

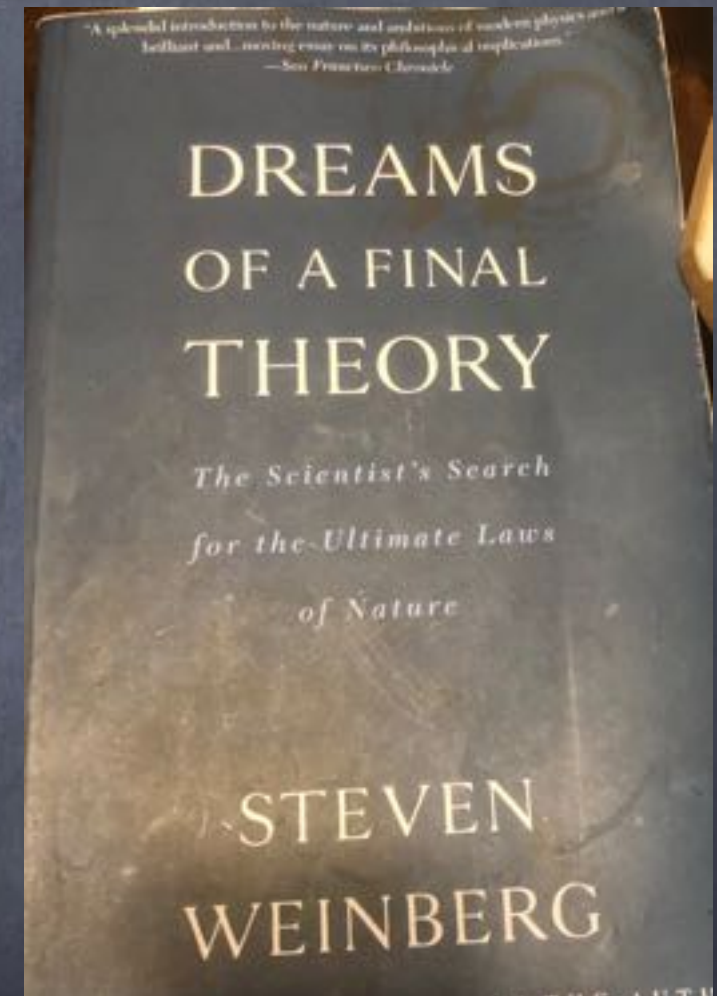
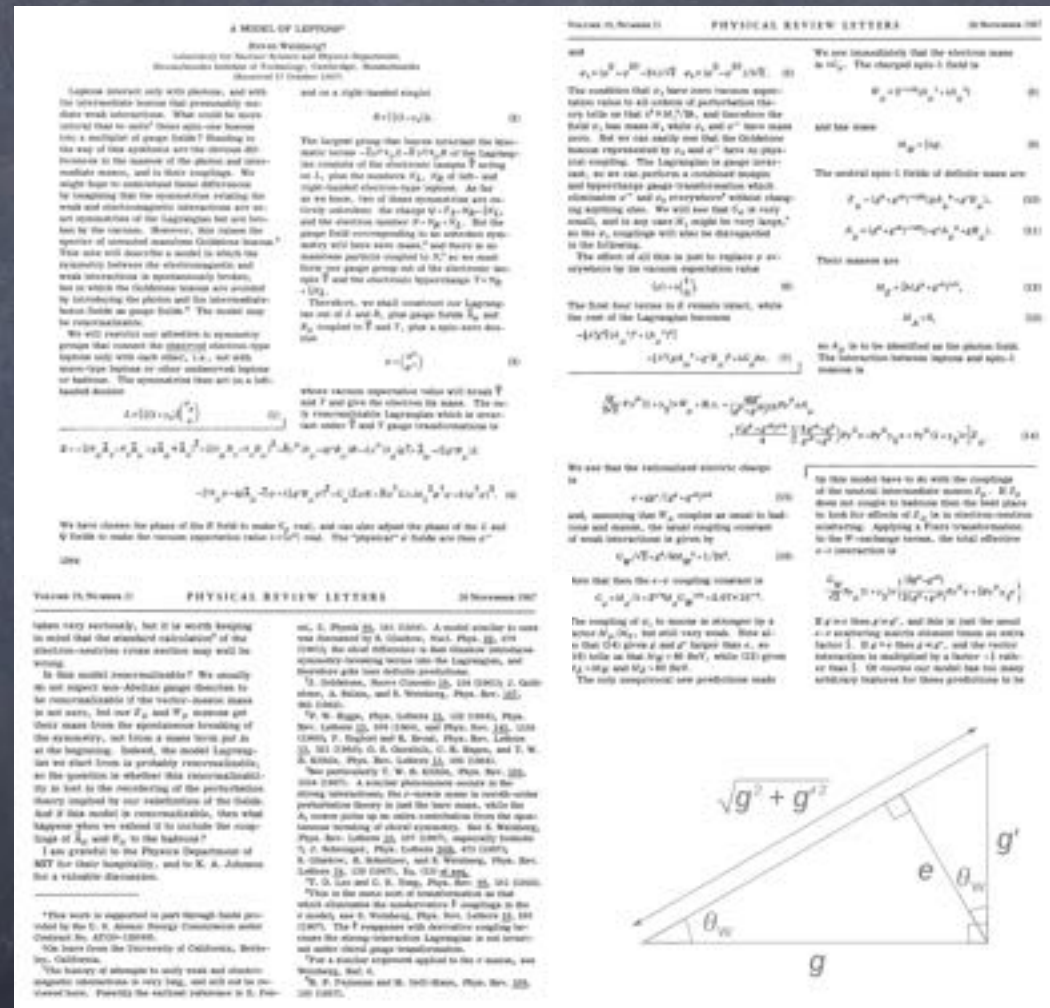
An Introduction to the Standard Model of Particle Physics

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2021 Pre SUSY Summer School · Beijing · Aug.9-13

In memory of Steven Weinberg

— one of founding fathers of the Standard Model of Particle Physics



Steven Weinberg
(1933–2021)

https://en.wikipedia.org/wiki/Steven_Weinberg

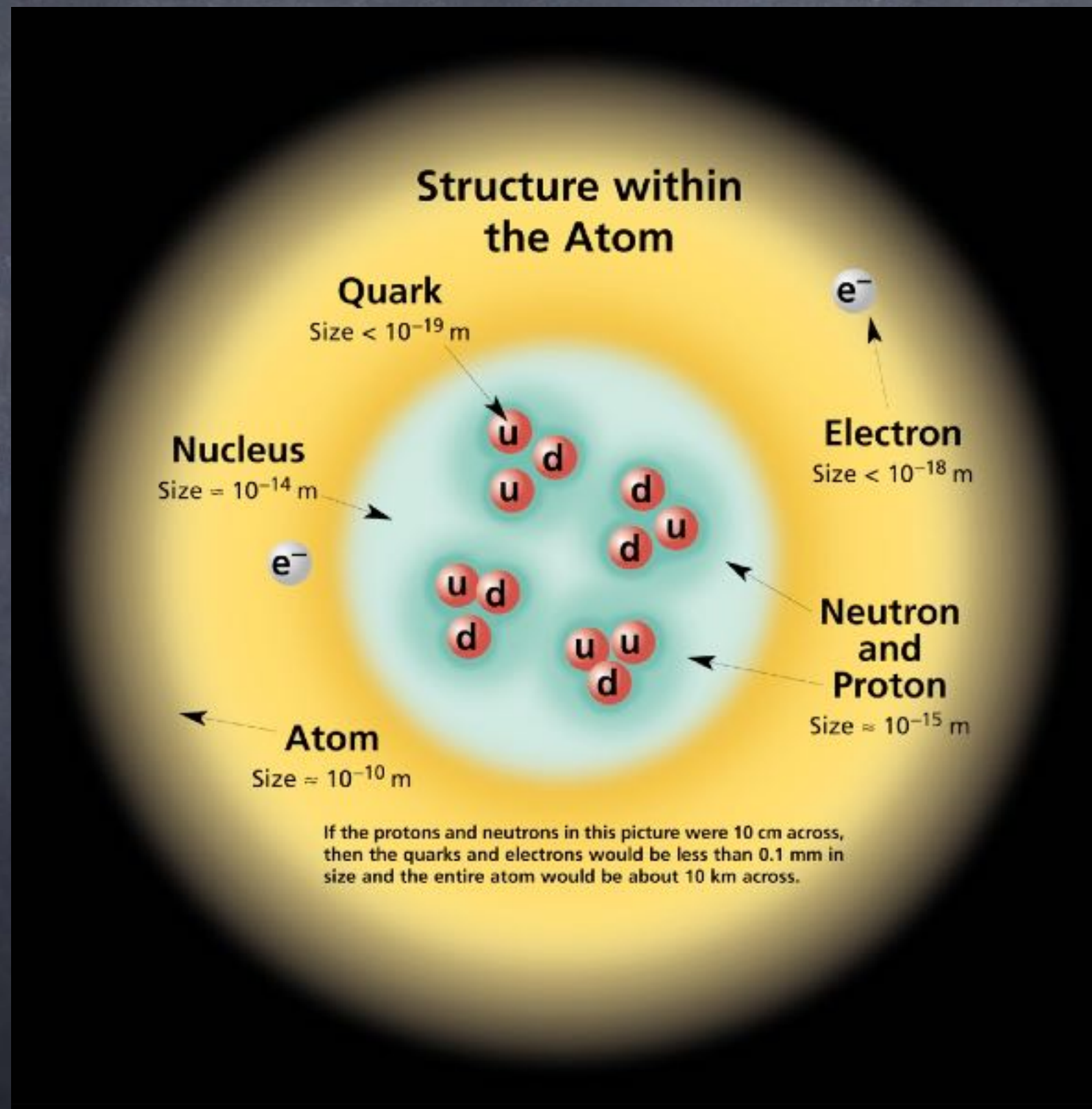
OUTLINE

- Brief introduction to (elementary) particle physics
- The Standard Model
 - Some basics of field theory: gauge symmetry, chiral fermion, Spontaneous symmetry breaking (SSB), gauge anomaly ...
 - Constructions of the SM Lagrangian (top-down)
 - Interactions after SSB
 - Successes of the SM
 - Unsatisfying issues in the SM
- Summary & Prospects

What is particle physics

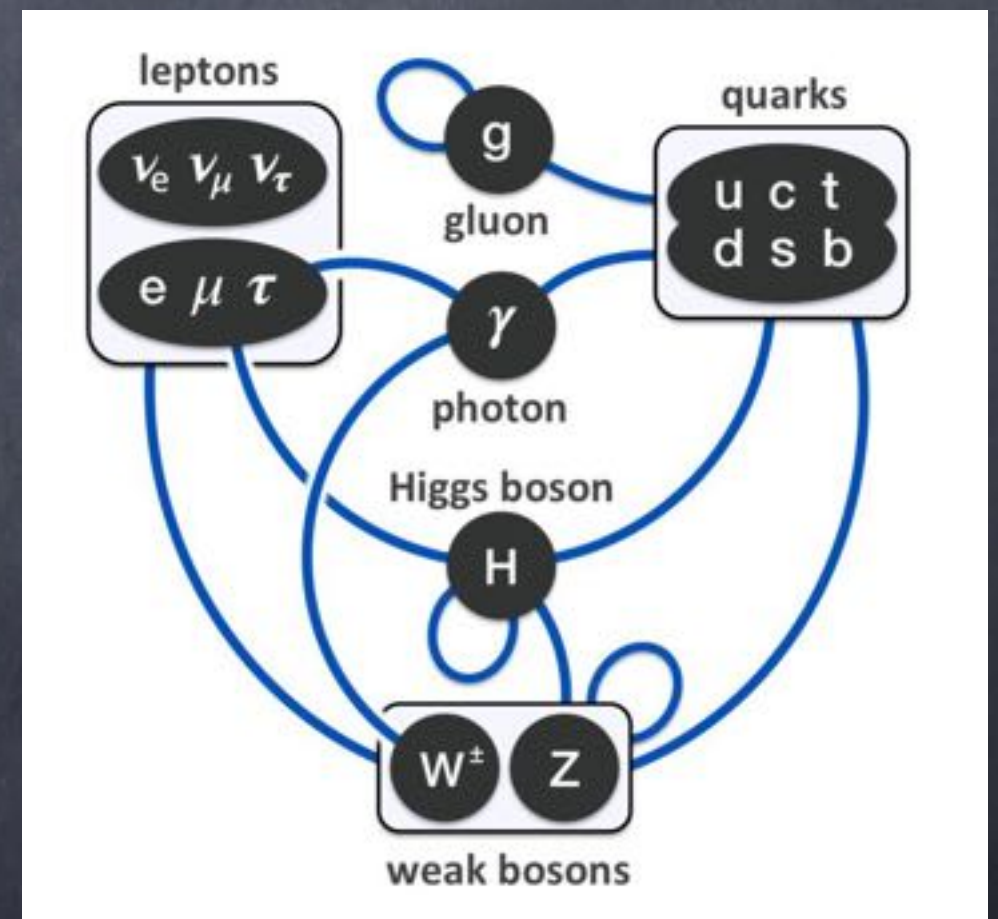
- Due to Wikipedia: Particle physics is a branch of physics that studies the nature of the particles that constitute matter and radiation.
- It probes the structure of our universe at the shortest scale. Due to the uncertainty principle, the short scale corresponds to high energy. Therefore, particle physics is also called high energy physics.
- Core topics:
 - What are elementary constitutions of matter?
 - How do they interact?
 - How do they form more complex matter states?
 - Can the descriptions about all particles and their fundamental interactions be unified into an ultimate theory? And if so, what is it?

The modern view of the subatomic world

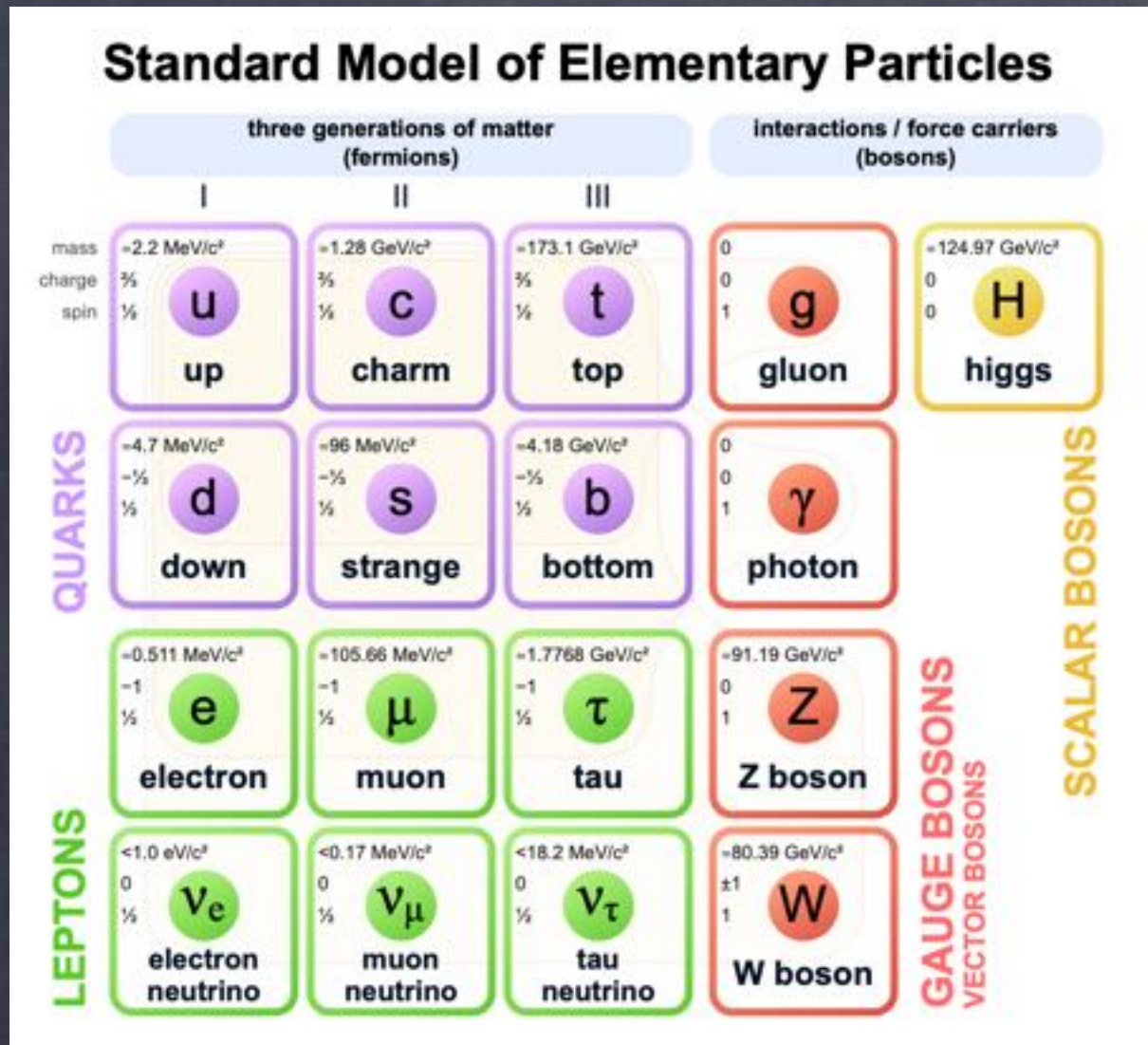


Four fundamental interactions

- **Electromagnetism:** The source of various forces in a common world ; interactions among electro-charged particles mediated by **photons**;
- **Strong interactions:** The bound between nucleons; interactions among quarks and gluons mediated by **gluons**;
- **Weak interactions:** β decays, nuclear fusion... mediators are **W/Z bosons**;
- **Gravity:** Interactions among everything (mediated by **gravitons?**); It is all about space-time and matters due to Einstein.



Fermions-matter constituents



FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-2) \times 10^{-9}$	0	u up	0.002	2/3
e electron	0.000511	-1	d down	0.005	-1/3
ν_M middle neutrino*	$(0.009-2) \times 10^{-9}$	0	c charm	1.3	2/3
μ muon	0.106	-1	s strange	0.1	-1/3
ν_H heaviest neutrino*	$(0.05-2) \times 10^{-9}$	0	t top	173	2/3
τ tau	1.777	-1	b bottom	4.2	-1/3

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Bosons-force carriers

- **spin-1/2**: 3 families of quarks and leptons;
- **spin-1**: gluons, W/Z bosons, photons;
- **spin-0**: Higgs (The God Particle)

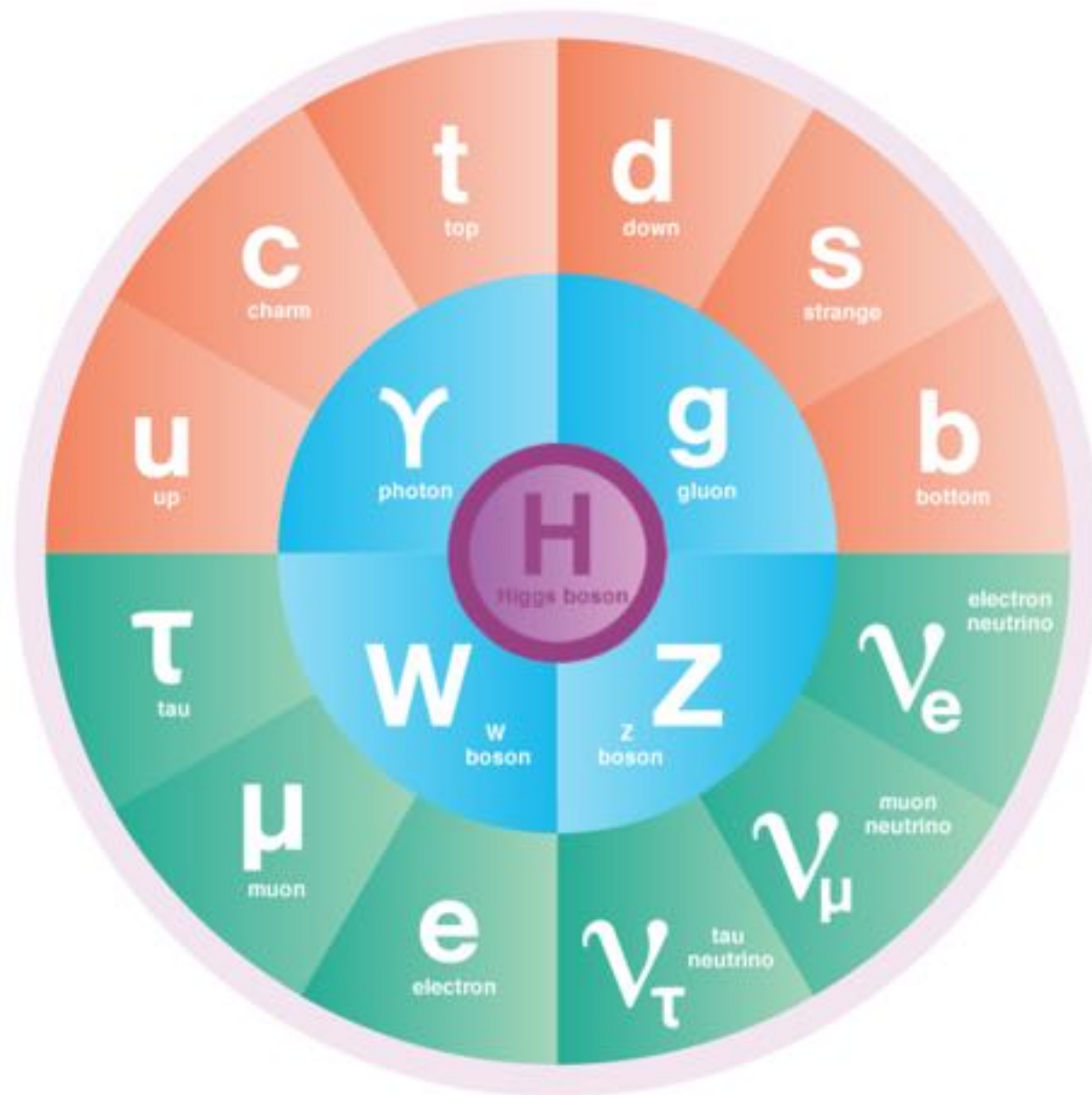
BOSONS force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge	Name	Mass GeV/c^2	Electric charge
γ photon	0	0	g gluon	0	0
W^-	80.39	-1	Higgs Boson spin = 0		
W^+ W bosons	80.39	+1			
Z^0 Z boson	91.188	0	Name	Mass GeV/c^2	Electric charge
			H Higgs	126	0

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The Standard Model

— the periodic table of elementary particles



The Standard Model of Particle Physics

HIGGS BOSON

Discovered in:
2012

Mass:
125.7 GeV

Discovered at:
CERN

Charge:
0

Spin:
0

About:

Discovered in 2012, the Higgs boson was the last missing piece of the Standard Model puzzle. It is a different kind of force carrier from the other elementary forces, and it gives mass to quarks as well as the W and Z bosons. Whether it also gives mass to neutrinos remains to be discovered.

Higgs bosons:
The last 🧩 for the SM

QUARKS LEPTONS BOSONS HIGGS BOSON

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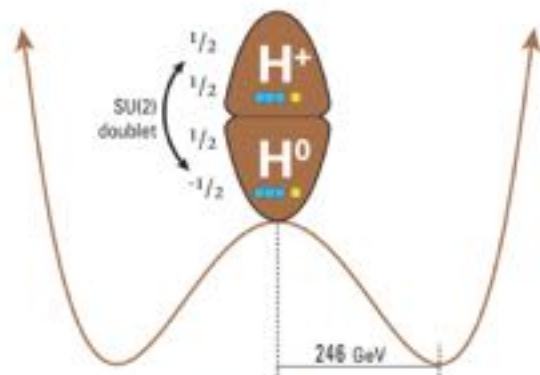
SM as a gauge field theory

- a spontaneously broken $SU_C(3) \times SU_L(2) \times U_Y(1)$ chiral gauge field theory to describe strong interactions (QCD), weak interactions & electromagnetism (EW);

The Standard Model of Particle Physics

Spin 0 (Higgs Boson)

Hypercharge $\rightarrow Y$
Weak Isospin $\rightarrow T_3$
Gauge boson coupling
Electric Charge $Q = Y + T_3$



Spin 1/2 (Fermions)

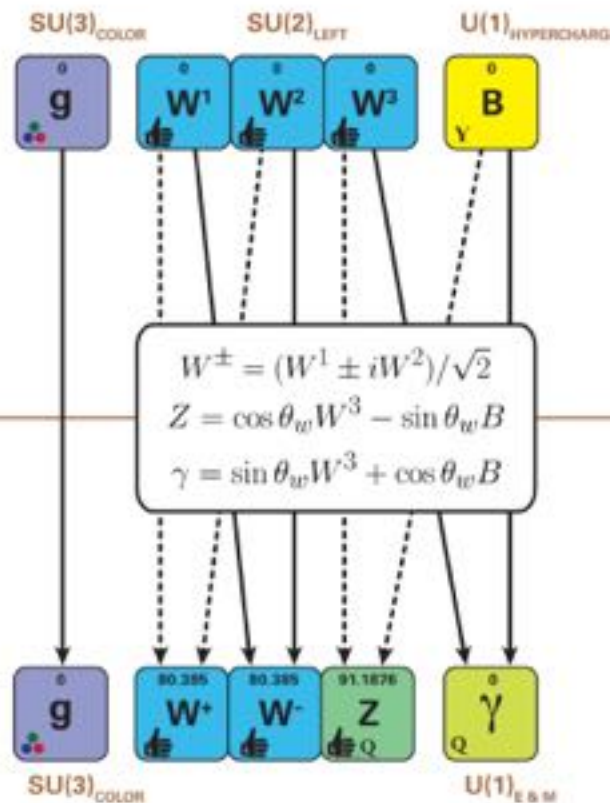
Generation
Hypercharge (L) $\rightarrow Y$
Weak Isospin (L) $\rightarrow T_3$
Gauge boson coupling
Electric Charge $Q = Y + T_3$

	1 st	2 nd	3 rd	
Quarks	$\begin{matrix} u \\ d \end{matrix}$	$\begin{matrix} c \\ s \end{matrix}$	$\begin{matrix} t \\ b \end{matrix}$	
Leptons	$\begin{matrix} \nu_e \\ e \end{matrix}$	$\begin{matrix} \nu_\mu \\ \mu \end{matrix}$	$\begin{matrix} \nu_\tau \\ \tau \end{matrix}$	

	1 st	2 nd	3 rd
Quarks	$\begin{matrix} u \\ d \end{matrix}$	$\begin{matrix} c \\ s \end{matrix}$	$\begin{matrix} t \\ b \end{matrix}$
Leptons	$\begin{matrix} \nu_e \\ e \end{matrix}$	$\begin{matrix} \nu_\mu \\ \mu \end{matrix}$	$\begin{matrix} \nu_\tau \\ \tau \end{matrix}$

Spin 1 (Gauge Bosons)

mass (GeV)
symbol
Fermion coupling
 $\sqrt{g^2 + g'^2}$
 θ_w
 e
 g'
 g



$$\begin{aligned}
 & -\frac{1}{2}\partial_\mu g_\nu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
 & \frac{1}{2}g_s^2 (q_i^\mu \gamma^\mu q_i^\mu) g_\mu^a + G^a \partial^2 G^a + g_s f^{abc} \partial_\mu G^a G^b G_\mu^c - \partial_\mu W_\nu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\nu A_\mu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2}M^2 \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \right. \\
 & \left. \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h - igc_w [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\nu^+ \partial_\mu W_\nu^- - W_\mu^+ \partial_\nu W_\nu^-) + Z_\mu^0 (W_\nu^+ \partial_\mu W_\nu^- - \\
 & W_\nu^- \partial_\mu W_\mu^+) - igc_w [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\mu W_\nu^- - \\
 & W_\nu^- \partial_\mu W_\mu^+) + A_\mu (W_\nu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+) - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \\
 & \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^-) + \\
 & g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\mu^- - A_\nu A_\mu W_\nu^+ W_\mu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha [H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
 & \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
 & gMW_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
 & W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}ig[W_\mu^+ (H\partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H\partial_\mu \phi^+ - \\
 & \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w} (Z_\mu^0 (H\partial_\mu \phi^0 - \phi^0 \partial_\mu H) - igc_w^2 MZ_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
 & igc_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig\frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
 & igc_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
 & \frac{1}{4}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)\phi^+ \phi^-] - \frac{1}{2}g^2 \frac{1}{c_w^2} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{1}{c_w^2} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{1}{2c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^4 s_w^2 A_\mu A_\mu \phi^+ \phi^- - e^4 (\gamma^\mu \partial_\mu + m_e^2) e^\lambda - e^4 \gamma^\mu \partial_\mu e^\lambda - \bar{u}_j^2 (\gamma^\mu \partial_\mu + m_u^2) u_j^\lambda - \\
 & \bar{d}_j^2 (\gamma^\mu \partial_\mu + m_d^2) d_j^\lambda + igc_w A_\mu [-(e^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^2 \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^2 \gamma^\mu d_j^\lambda)] + \\
 & \frac{ig}{2c_w} Z_\mu^0 [(\bar{u}_j^2 \gamma^\mu (1 + \gamma^5) u_j^\lambda) + (e^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^2 \gamma^\mu (\frac{1}{3}s_w^2 - \\
 & 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^2 \gamma^\mu (1 - \frac{2}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
 & (\bar{u}_j^2 \gamma^\mu (1 + \gamma^5) C_{\lambda\mu} d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^2 C_{\lambda\mu}^1 \gamma^\mu (1 + \\
 & \gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}} \frac{m_h^2}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
 & \frac{g}{2} \frac{m_h^2}{M} [H(\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^2 (\bar{u}_j^2 C_{\lambda\mu} (1 - \gamma^5) d_j^\lambda) + \\
 & m_u^2 (\bar{u}_j^2 C_{\lambda\mu} (1 + \gamma^5) d_j^\lambda) + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^2 (\bar{d}_j^2 C_{\lambda\mu}^1 (1 + \gamma^5) u_j^\lambda) - m_u^2 (\bar{d}_j^2 C_{\lambda\mu}^1 (1 - \\
 & \gamma^5) u_j^\lambda) - \frac{g}{2} \frac{m_h^2}{M} H(\bar{u}_j^2 u_j^\lambda) - \frac{g}{2} \frac{m_h^2}{M} H(\bar{d}_j^2 d_j^\lambda) + \frac{ig}{2} \frac{m_h^2}{M} \phi^0 (\bar{u}_j^2 \gamma^5 u_j^\lambda) - \\
 & \frac{ig}{2} \frac{m_h^2}{M} \phi^0 (\bar{d}_j^2 \gamma^5 d_j^\lambda) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \\
 & \frac{M^2}{c_w^2}) X^0 + Y \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^+ X^-) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igc_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w} \bar{X}^0 X^0 H] + \\
 & \frac{1-2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^- - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
 & igM s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$

Lagrangian standard model
Courtesy of T.D. Gutierrez

The making of the SM

– Brief History of Particle Physics (Efforts from theorists and experimentalists)

1970's

- Rise of the Standard Model theory (Electroweak and QCD)
- Discovery of J/Ψ (charm quark) in 1974 , **November Revolution!**
- Discovery of τ lepton, bottom quark, gluon

1980's

- Discovery of weak W^\pm and Z^0 bosons

1990's

- Discovery of top quark
- $N_\nu = 3$, great success of the Standard Model (gauge theory)
- Discovery of neutrino oscillation

the 21 st century

- Discovery of CP violation in B decays, success of KM theory
- Discovery of the Higgs particle in 2012
- Find the TeV scale new physics. \Rightarrow **New Revolution ?**



???

How to build a gauge theory

- U(1) case: QED & Maxwell Equations are invariant under local/gauge transformations

$$\Psi(x) \rightarrow e^{-i\alpha(x)}\Psi(x), \quad eA_\mu(x) \rightarrow eA_\mu(x) + \partial_\mu\alpha(x).$$

Construct the covariant derivative: $D_\mu\Psi = (\partial_\mu + ieA_\mu)\Psi$,

so that $D_\mu\Psi \rightarrow e^{-i\alpha(x)}D_\mu\Psi$. The gauge invariant Lagrangian density is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\not{D} - m)\Psi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

- SU(2) case: Yang-Mills theory (1954')
a theory of nucleons & pions with local isospin symmetry

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^i F^{i\mu\nu} \\ & + \bar{\Psi}(i\not{D} - m_N)\Psi \\ & + \frac{1}{4}\text{tr}[D_\mu\pi D^\mu\pi - m_\pi^2\pi^2] \\ & + ig_{\pi N}\bar{\Psi}\gamma^5\pi\Psi, \end{aligned}$$

$$\begin{aligned} F_{\mu\nu}^i &\equiv \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g\epsilon^{ijk}A_\mu^j A_\nu^k, \\ \text{with } D_\mu\Psi &\equiv (\partial_\mu - igA_\mu)\Psi, \\ D_\mu\pi &= \partial_\mu\pi(x) - g[A_\mu(x), \pi(x)]. \end{aligned}$$



is invariant under transformations

$$\left\{ \begin{aligned} \Psi(x) &\equiv \begin{pmatrix} p(x) \\ n(x) \end{pmatrix} \rightarrow U(x)\Psi(x), \quad U(x) \equiv e^{i\alpha^i(x)\frac{\sigma^i}{2}}, \\ \pi(x) &\equiv \pi^i(x)\sigma^i \rightarrow U(x)\pi(x)U^\dagger(x) \\ gA_\mu(x) &\equiv gA_\mu^i(x)\sigma^i/2 \\ &\rightarrow U(x)gA_\mu(x)U^\dagger + iU(x)\partial_\mu U^\dagger(x). \end{aligned} \right.$$

Some properties of gauge theories

1. A gauge boson corresponds to a gauge symmetry;
2. Gauge symmetry forbids mass terms for gauge fields! As long as the gauge symmetry is preserved, the corresponding **gauge boson is massless!**
3. For $U(1)$ gauge theory, the gauge boson does not have self-interactions; For non-abelian gauge theory, the gauge bosons can interact with each other. This very difference between $U(1)$ gauge theories and non-abelian gauge theories leads to **the asymptotic freedom** of non-abelian gauge theories which is essential for building a gauge theory of strong interaction/a grand unified theory (GUT).
4. The gauge symmetries can be broken either spontaneously or by quantum anomalies if we introduce the chiral fermionic multiplets inconsistently. Some gauge symmetries of SM are spontaneously broken, and corresponding gauge bosons become massive. However, SM is free of various quantum anomalies due to the chiral fermions.

Spontaneous symmetry breaking

- **SSB:** In a quantum mechanical system, a symmetry of the action is not a symmetry of the ground state (in other words, the ground states are degenerate), then the symmetry is broken spontaneously.
- **Goldstone Theorem:** Every spontaneously broken global continuous symmetry corresponds to a massless scalar (Nambu-Goldstone boson) in the theory. (1960')

First 2 rigorous proofs given by Goldstone, Salam and Weinberg (1962')

Eg. Expanding the scalar potential $V(\phi)$ around the minimum (vacuum) ϕ_0 ,

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a(\phi - \phi_0)^b m_{ab}^2 + \dots \quad \text{with } m_{ab}^2 \equiv \left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b} \right)_{\phi=\phi_0} \quad \text{being}$$

the mass square matrix. $V(\phi)$ is invariant under linear transformation

$\phi^a \rightarrow \phi^a + \alpha \Delta^a(\phi)$, therefore $\Delta^a(\phi) \frac{\partial V}{\partial \phi^a} = 0$. Differentiate it again, we get

$$0 = \left(\frac{\partial \Delta^a}{\partial \phi^b} \right)_{\phi_0} \left(\frac{\partial V}{\partial \phi^a} \right)_{\phi_0} + \Delta^a(\phi_0) \left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b} \right)_{\phi_0} = \Delta^a(\phi_0) \left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b} \right)_{\phi_0}.$$

when $\Delta^a(\phi_0) \neq 0$ (which means vacuum is not invariant under the symmetry transformation of the potential), m^2 has an eigenvalue 0, which implies a massless scalar boson (Goldstone) in the theory.

(Anderson)–(Brout–Englert)–Higgs–(Guralnik–Hagen–Kibble)–('t Hooft) mechanism

Every spontaneously broken local continuous symmetry (gauge symmetry) corresponds to a massive gauge boson in the theory. (1962')

Eg. The scalar bosons in previous slide couple to gauge bosons through the minimal coupling, i.e. through the covariant derivatives

$$\frac{1}{2} \left(D_\mu \phi^i \right)^2 = \frac{1}{2} \left(\partial_\mu \phi^i \right)^2 + g A_\mu^a \left(\partial_\mu \phi^i T_{ij}^a \phi^j \right) + \frac{1}{2} g^2 A_\mu^a A_\mu^b \left(T^a \phi \right)^i \left(T^b \phi \right)^i$$

The gauge symmetries are spontaneously broken by the vacuum of the scalar fields $\langle \phi^i \rangle_{\text{VAC}} = (\phi_0)^i$, if $T^a \phi_0 \neq 0$. There appears the mass term for the gauge bosons

$$\Delta \mathcal{L} = \frac{1}{2} m_{ab}^2 A_\mu^a A^{b,\mu}, \quad m_{ab}^2 = g^2 \left(T^a \phi_0 \right)^i \left(T^b \phi_0 \right)^i .$$

Remarks:

1. The gauge bosons corresponding to unbroken symmetries ($T^a \phi_0 \neq 0$) remain massless!
2. The SSB gauge symmetries are not really broken, rather hidden!
3. The massless Goldstones become the longitudinal component of the massive gauge bosons! Degree of freedom does not change!

1 massless scalar + 2 massless spin-1 D.O.F. = 0 scalar + 3 massive spin-1 D.O.F.

(First raised by GHK in 1964')

Chiral fermions

- 4-component spinor in Weyl representation:

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \Psi_L \equiv \frac{1 - \gamma^5}{2} \Psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \Psi_R \equiv \frac{1 + \gamma^5}{2} \Psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}.$$

- Lorentz transformations:

$$\psi_L \rightarrow \left(1 - i\alpha^i \frac{\sigma^i}{2} - \beta^i \frac{\sigma^i}{2} \right) \psi_L, \quad \psi_R \rightarrow \left(1 - i\alpha^i \frac{\sigma^i}{2} + \beta^i \frac{\sigma^i}{2} \right) \psi_R$$

$$c\psi_R^* \rightarrow \left(1 - i\alpha^i \frac{\sigma^i}{2} - \beta^i \frac{\sigma^i}{2} \right) c\psi_R^*, \quad c = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

- Mass terms:

- Dirac mass**—couplings between left-handed and right-handed 2-component spinors

$$-m_D (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L)$$

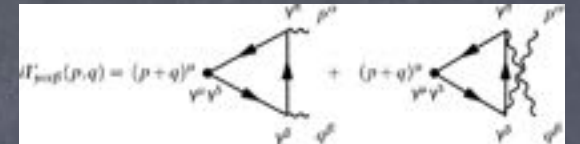
- Majorana mass** — couplings between 2-component spinors with same chirality

$$-m_M^L \Psi_L^T C \Psi_L - m_M^R \Psi_R^T C \Psi_R + \text{h.c.}$$

Chiral anomaly

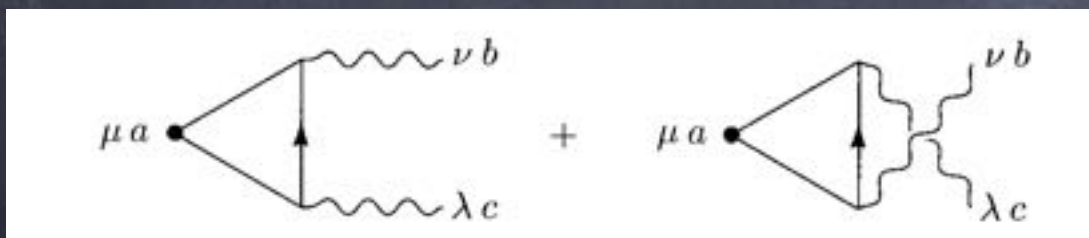
- Adler-Bell-Jakiw anomaly:** The Noether current corresponding to a global chiral symmetry of a classical theory of chiral fermions is not conserved any more quantum mechanically, **when chiral fermions couple to gauge bosons.**

ABJ anomaly equation: $\partial_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi = \frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$



However, this is normal and useful. It leads to $\pi^0 \rightarrow \gamma\gamma$ and possible **baryon number non-conservation of the SM.**

- Gauge anomaly:** If the chiral Noether current is coupled to a gauge boson, the corresponding **gauge symmetry may be broken quantum mechanically!**



$$\propto \text{tr}[T^a \{T^b, T^c\}] = \sum_r \text{tr}[T_r^a \{T_r^b, T_r^c\}]$$

$$= \sum_r D_r^{abc} = D^{abc}$$

- Gravity anomaly:** If the chiral fermions couple to gravity, there will be new anomaly $\partial_\mu (\bar{\chi} T \gamma^\mu \chi) \approx \text{Tr}[T] \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}^{\kappa\lambda}$ **which breaks the corresponding gauge symmetry as well!**

Gauge anomaly cancelation

- All anomalies which break the gauge symmetries should be cancelled! The gauge groups or/and representations of chiral fermions should be chosen wisely!
- Gauge anomaly cancellation : $\sum_r D_r^{abc} = 0$, $\sum_r \text{tr} T_r = 0$.
- For (pseudo)-real representation: $D_r^{abc} = 0$ automatically!
- Group theory:
 - Compact semi-simple groups with only pseudo-real or real reprs (self-conjugate reprs): $SU(2)$, $SO(2n+1)$, $SO(4n)$ ($n \geq 2$), $USp(2n)$, G_2 , F_4 , E_7 , E_8 ;
 - Compact semi-simple groups with non-self-conjugate reprs but $D_r^{abc} \equiv 0$: $SO(4n+2)$ ($n \geq 2$), E_6 ;
 - ONLY $SU(n)$ ($n \geq 3$) and $U(1)$ gauge theories can have gauge anomaly!
 - Global anomaly (E.Witten 1982'): For $SU(2)$ chiral gauge theory, the number of pseudo-real reprs of chiral fermions must be even to avoid anomaly.

Fields in SM

- **Representations:** $(\text{SU}(3) : R_3, \text{SU}(2) : R_2)_{\text{Hypercharge}:Y}$
- **Quarks:** $Q_L^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L = (3, 2)_{+\frac{1}{6}}, u_R^i = (3, 1)_{+\frac{2}{3}}, d_R^i = (3, 1)_{-\frac{1}{3}} ;$
- **Leptons:** $E_L^i = \begin{pmatrix} \nu^i \\ e^i \end{pmatrix}_L = (1, 2)_{-\frac{1}{2}}, e_R^i = (1, 2)_{-1} ;$
 Note: No right-handed neutrinos ν_R^i !
- **Gauge potential:** G_μ^A ($A = 1, 2, \dots, 8$), W_μ^a ($a = 1, 2, 3$), B_μ ; 3 gauge couplings: g_s, g, g' ;
- **Higgs field:** $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = (1, 2)_{+\frac{1}{2}} ;$
- **Covariant derivatives:** $D_\mu = \partial_\mu - ig_s G_\mu^A T_3^A - ig W_\mu^a T_2^a - ig' Y B_\mu$

Covariant derivatives

$$\left\{ \begin{array}{l} D_\mu Q_L = \left(\partial_\mu - ig_s \frac{\lambda^A}{2} G_\mu^A - ig \frac{\sigma^a}{2} W_\mu^a + i \frac{1}{6} g' B_\mu \right) Q_L, \\ D_\mu E_L = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - i \frac{1}{2} g' B_\mu \right) E_L, \\ D_\mu u_R = \left(\partial_\mu - ig_s \frac{\lambda^A}{2} G_\mu^A - i \frac{2}{3} g' B_\mu \right) u_R, \\ D_\mu d_R = \left(\partial_\mu - ig_s \frac{\lambda^A}{2} G_\mu^A + i \frac{1}{3} g' B_\mu \right) d_R, \\ D_\mu e_R = \left(\partial_\mu + i \frac{1}{2} g' B_\mu \right) e_R, \\ D_\mu H = \left(\partial_\mu + i \frac{1}{2} g' B_\mu \right) H, \end{array} \right.$$

8 Gell-Mann matrices:

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda^2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \lambda^3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda^4 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda^5 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda^6 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ \lambda^7 &= \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, & \lambda^8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \\ \left[\frac{\lambda^A}{2}, \frac{\lambda^B}{2} \right] &= if^{ABC} \frac{\lambda^C}{2}, \quad A, B, C = 1, 2, \dots, 8. \end{aligned}$$



3 Pauli matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$

$$\left[\frac{\sigma^a}{2}, \frac{\sigma^b}{2} \right] = i\epsilon^{abc} \frac{\sigma^c}{2}, \quad a, b, c = 1, 2, 3.$$



SM Lagrangian

$$\left\{ \begin{array}{l} \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \\ \mathcal{L}_{\text{gauge}} = -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}, \\ \mathcal{L}_{\text{quark}} = \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_R^i \not{D} u_R^i + \bar{d}_R^i \not{D} d_R^i, \\ \mathcal{L}_{\text{lepton}} = \bar{E}_L^i \not{D} E_L^i + \bar{e}_R^i \not{D} e_R^i, \\ \mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger D^\mu H - V(H, H^\dagger), \\ \mathcal{L}_{\text{Yukawa}} = Y_u^{ij} \bar{Q}_L^i u_R^j \widetilde{H} + Y_d^{ij} \bar{Q}_L^i d_R^j H + Y_e^{ij} \bar{E}_L^i e_R^j H + \text{h.c.}, \\ V(H, H^\dagger) = -\mu^2 H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 = \frac{\lambda}{4} \left(H^\dagger H - \frac{v^2}{2} \right) + \dots \end{array} \right.$$



Remarks:

1. No mass terms for fermions & gauge bosons!
2. Most of parameters are from Higgs-sector and Yukawa couplings!

Gauge anomaly cancellation in SM

- Convert the right-handed chiral fermions to left-handed ones
 $u_R^{i,c} = Cu_R^* = (\bar{3}, 1)_{-2/3}$, $d_R^{i,c} = Cd_R^* = (\bar{3}, 1)_{+1/3}$, $e_R^{i,c} = Ce_R^* = (1, 1)_{+1}$,
- SU(3)-SU(3)-SU(3): Repr. of SU(3) sector $3 \oplus 3 \oplus \bar{3} \oplus \bar{3} \oplus 1 \oplus 1$ is real, $D^{ABC} \equiv 0$;
- SU(3)-SU(3)-SU(2): $D^{ABc} \propto \text{tr}(\sigma^c/2) = 0$;
- SU(3)-SU(2)-SU(2): $D^{Abc} \propto \text{tr}(\lambda^A/2) = 0$;
- SU(2)-SU(2)-SU(2): $D^{abc} \equiv 0$;
- SU(2) global anomaly cancellation: 4 SU(2) doublets (3 quark doublets and 1 lepton doublet) in each generation in SM;
- graviton-graviton-SU(3): $\text{tr}(\lambda^A/2) = 0$;
- graviton-graviton-SU(2): $\text{tr}(\sigma^c/2) = 0$;

Gauge anomaly cancellation: U(1) sector

- SU(3)-SU(3)-U(1):

$$\sum_{\text{quarks}} Y = 2(+1/6) - 2/3 + 1/3 = 0.$$

- SU(2)-SU(2)-U(1):

$$\sum_{\text{doublets}} Y = 3(+1/6) - 1/2 = 0.$$

- U(1)-U(1)-U(1):

$$\sum Y^3 = 6(+1/6)^3 + 3(-2/3)^3 + 3(1/3)^3 + 2(-1/2)^3 + 1^3 = 0.$$

- graviton-graviton-U(1):

$$\sum Y = 6(+1/6) + 3(-2/3) + 3(+1/3) + 2(-1/2) + 1 = 0$$

SSB in SM

Minimizing the potential $V(H, H^\dagger)$, $\langle H^\dagger H \rangle_{\text{vac}} = v^2/2$. The vacuum expectation value (VEV) of the Higgs field (due to the freedom given by gauge invariance or in so-called unitary gauge)

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with } v \simeq 246 \text{ GeV}.$$

We see that $T^a \langle H \rangle \neq 0$, ($T^a = \sigma^a/2$), $Y \langle H \rangle \equiv \frac{1}{2} \langle H \rangle \neq 0$, but $(T^3 + Y) \langle H \rangle = 0$.

This means that the gauge symmetry $\text{SU}_L(2) \otimes \text{U}_Y(1)$ is spontaneously broken into $\text{U}_Q(1)$ where

$$Q \equiv T^3 + Y \quad (\text{Gell-Mann-Nishijima})$$

is exactly the electric charge in unit of an elementary charge. We have

$$\begin{cases} Q_u = +1/2 + 1/6 = 0 + 2/3 = +2/3, \\ Q_d = -1/2 + 1/6 = 0 - 1/3 = -1/3, \\ Q_e = -1/2 - 1/2 = 0 - 1 = -1, \\ Q_\nu = +1/2 - 1/2 = 0 - 0 = 0. \end{cases}$$

Remark: If we introduce right-handed neutrino which must be $\text{SU}(2)$ singlet, it will have 0 hyper-charge too. Therefore, the right-handed neutrino must be SM singlet.

Gauge bosons after SSB in SM

Introduce: $W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (W_{\mu}^1 \mp iW_{\mu}^2)$, $T^{\pm} = T^1 \pm iT^2$, and

$$\begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_{\mu}^3 \\ B_{\mu} \end{pmatrix}, \text{ with } \sin \theta_W = g' / \sqrt{g^2 + g'^2}$$

where θ_W is the Weinberg angle, the covariant derivative becomes

$$D_{\mu} = \partial_{\mu} - igW_{\mu}^a T^a - ig'YB_{\mu}$$

$$= \partial_{\mu} - i\frac{g}{\sqrt{2}} (W^{+}T^{+} + W^{-}T^{-}) - ig \cos \theta_W (T^3 - \tan^2 \theta_W Y) Z_{\mu} - ieQA_{\mu},$$

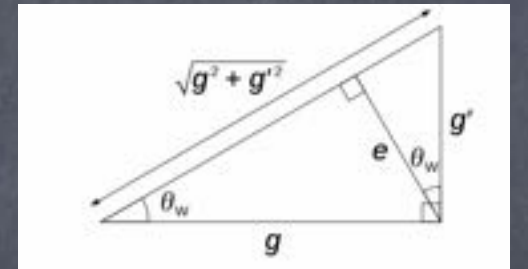
where $e = g \sin \theta_W$, and $Q = T^3 + Y$. In the unitary gauge, we parameterize

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

$$(D_{\mu}H)^{\dagger} (D^{\mu}H) = \frac{1}{2} \partial_{\mu} h(x) \partial^{\mu} h(x) + \frac{g^2}{4} (v + h(x))^2 \left(W_{\mu}^{+} W^{-\mu} + \frac{1}{2 \cos^2 \theta_W} Z_{\mu} Z^{\mu} \right).$$

Hence the masses for gauge bosons are

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2) v^2}{4}, \quad M_A^2 = 0.$$



Fermions after SSB in SM

- Mass terms:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= \bar{Q}_L y_d d_R H + \bar{Q}_L y_u u_R \widetilde{H} + \bar{E}_L y_e \ell_R H + \text{h.c.} \\ &= (\bar{d}_L y_d d_R + \bar{u}_L y_u u_R + \bar{\ell}_L y_e \ell_R) \frac{v + h(x)}{\sqrt{2}} + \text{h.c.}\end{aligned}$$

Diagonalizing the Yukawa couplings, we can get the Dirac mass terms of fermions

$$\mathcal{L}_{\text{mass}} = - \sum_f m_f \bar{f} f, \quad \text{with} \quad m_f = - y_f v / \sqrt{2}.$$

- Couplings between fermions and gauge bosons:

$$\mathcal{L}_{\text{current}} = g \left(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu} + Z_\mu J_Z^\mu \right) + e A_\mu J_{\text{em}}^\mu,$$

- Charge current:

$$J_W^{+\mu} = \frac{1}{\sqrt{2}} (\bar{\nu}_L \gamma^\mu V_{\text{CKM}} \ell_L + \bar{u}_L \gamma^\mu d_L), \quad J_W^{-\mu} = \frac{1}{\sqrt{2}} (\bar{\ell}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu V_{\text{CKM}}^\dagger u_L),$$

- Neutral current:

$$J_Z^\mu = \frac{1}{\cos \theta_W} \sum_{f=Q_L, u_R, d_R, E_L, \ell_R} \bar{f} \gamma^\mu (T^3 - Q \sin^2 \theta_W) f,$$

- Electro-magnetic current: $J_{\text{em}}^\mu = - \bar{\ell} \gamma^\mu \ell + \frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d$

CKM matrix: source of CP violation in SM

- Charged current interaction:

CKM matrix describes the mixing among the down-type quarks

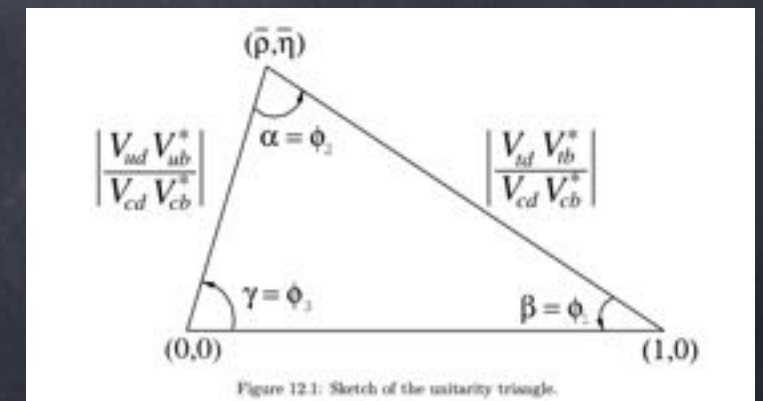
$$\frac{g}{\sqrt{2}} (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu W_\mu^+ V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}, \quad V_{\text{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Kobayashi-Maskawa Theory:** # of CPV phases = $n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$, $n = \#$ of generations. 3 generations allow 1 complex phase \Rightarrow CPV

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- The unitarity of CKM matrix can be represented as a **unitary triangle** in complex plane.

More information from lectures by Prof. C.D. Lü in this school



Quantized SM Lagrangian:

SM=QCD+EW

1. QCD: strong interactions among quarks and gluons (C.F. Qiao's lect.)
2. gauge interactions among W, Z, Higgs bosons
3. charge-current and neutral-current interactions among W/Z and fermions
4. Yukawa couplings among fermions and Higgs bosons
5. ghosts due to the Faddeev-Popov quantization

$$\begin{aligned}
 & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
 & \frac{1}{2}ig_s^2(\bar{q}_i^c \gamma^\mu q_i^c)g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu G^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2\epsilon_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2\epsilon_w^2} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \right. \\
 & \left. \frac{2\lambda}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h - igc_w [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\mu^- W_\nu^+) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\mu W_\mu^- - \\
 & W_\nu^- \partial_\mu W_\mu^+)] - ig s_w [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - A_\nu (W_\mu^+ \partial_\mu W_\mu^- - \\
 & W_\mu^- \partial_\mu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \\
 & \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + \\
 & g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w [A_\mu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\mu^- W_\nu^+) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha [H^4 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
 & \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
 & gM W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{\epsilon_w^2} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
 & W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g[W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \\
 & \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{\epsilon_w^2} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{1-2c_w^2}{2\epsilon_w^2} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
 & ig s_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2\epsilon_w^2} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
 & ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
 & \frac{1}{4}g^2 \frac{1}{\epsilon_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)\phi^+ \phi^-] - \frac{1}{2}g^2 \frac{c_w^2}{\epsilon_w^2} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{c_w^2}{\epsilon_w^2} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{c_w^2}{\epsilon_w^2} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^4 s_w^2 A_\mu A_\mu \phi^+ \phi^- - e^4 (\gamma^\mu \partial + m_e^2) e^\lambda - e^\lambda \gamma^\mu \partial e^\lambda - \bar{u}_j^2 (\gamma^\mu \partial + m_u^2) u_j^2 - \\
 & \bar{d}_j^2 (\gamma^\mu \partial + m_d^2) d_j^2 + ig s_w A_\mu [-(e^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^2 \gamma^\mu u_j^2) - \frac{1}{3}(\bar{d}_j^2 \gamma^\mu d_j^2)] + \\
 & \frac{ig}{4\epsilon_w} Z_\mu^0 [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^2 \gamma^\mu (\frac{4}{3}s_w^2 - \\
 & 1 - \gamma^5) u_j^2) + (\bar{d}_j^2 \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^2)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
 & (\bar{u}_j^2 \gamma^\mu (1 + \gamma^5) C_{\lambda\mu} d_j^2)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^2 C_{\lambda\mu}^1 \gamma^\mu (1 + \\
 & \gamma^5) u_j^2)] + \frac{ig}{2\sqrt{2}} \frac{m_h^2}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
 & \frac{g}{2} \frac{m_h^2}{M} [H(\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^2 (\bar{u}_j^2 C_{\lambda\mu} (1 - \gamma^5) d_j^2) + \\
 & m_u^2 (\bar{u}_j^2 C_{\lambda\mu} (1 + \gamma^5) d_j^2)] + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^2 (\bar{d}_j^2 C_{\lambda\mu}^1 (1 + \gamma^5) u_j^2) - m_u^2 (\bar{d}_j^2 C_{\lambda\mu}^1 (1 - \\
 & \gamma^5) u_j^2)] - \frac{g}{2} \frac{m_h^2}{M} H (\bar{u}_j^2 u_j^2) - \frac{g}{2} \frac{m_h^2}{M} H (\bar{d}_j^2 d_j^2) + \frac{ig}{2} \frac{m_h^2}{M} \phi^0 (\bar{u}_j^2 \gamma^5 u_j^2) - \\
 & \frac{ig}{2} \frac{m_h^2}{M} \phi^0 (\bar{d}_j^2 \gamma^5 d_j^2) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \\
 & \frac{M^2}{\epsilon_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \\
 & \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \\
 & \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{\epsilon_w^2} \bar{X}^0 X^0 H] + \\
 & \frac{1-2c_w^2}{2\epsilon_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2\epsilon_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
 & igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$

Small summary of the theoretical prospects of the SM

- Renormalizable theory!
- Gauge SSB: $SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \rightarrow U_{\text{em}}(1)$;
- Gauge anomaly free;
- ONLY 18 parameters:
 - 3 gauge couplings: $g_1 = g'$, $g_2 = g$ and $g_3 = g_s$;
 - 9 fermion masses: $m_u, m_d, m_s, m_c, m_b, m_t, m_e, m_\mu, m_\tau$;
 - 4 CKM matrix parameters: λ, A, ρ, η (Wolfenstein parameterization);
 - 1 Higgs vacuum expectation value (VEV): v ; (related to m_W and m_Z)
 - 1 Higgs mass: m_H ; (related to Higgs self-coupling λ and VEV v)
- Global symmetries:
 - P and C are maximally violated in weak interactions;
 - CP and T can be violated;
 - $U_B(1)$ and $U_L(1)$ global symmetry: SM (before and after SSB) is invariant under $q \rightarrow e^{-i\alpha/3}q$ and $\ell \rightarrow e^{-i\beta}\ell, \nu \rightarrow e^{-i\beta}\nu$
 \Rightarrow Baryon number and lepton number conservation

One page summary of the world

Gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Particle content

MATTER				HIGGS		GAUGE	
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2})_{1/3}$	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{-1}$	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_1$	B	$(\mathbf{1}, \mathbf{1})_0$
u_R^c	$(\bar{\mathbf{3}}, \mathbf{1})_{-4/3}$	e_R^c	$(\mathbf{1}, \mathbf{1})_2$			W	$(\mathbf{1}, \mathbf{3})_0$
d_R^c	$(\bar{\mathbf{3}}, \mathbf{1})_{2/3}$	ν_R^c	$(\mathbf{1}, \mathbf{1})_0$			G	$(\mathbf{8}, \mathbf{1})_0$

Lagrangian
(Lorentz + gauge + renormalizable)

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^\alpha G^{\alpha\mu\nu} + \dots \bar{Q}_k \not{D} Q_k + \dots (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{4!} (H^\dagger H)^2 + \dots Y_{k\ell} \bar{Q}_k H (u_R)_\ell$$

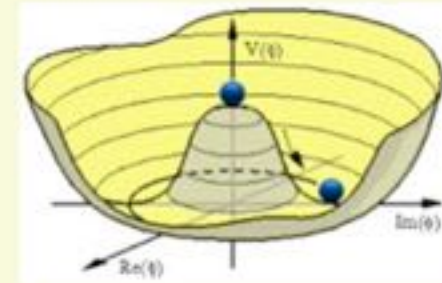
SSB

$$H \rightarrow H' + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

$$B, W^3 \rightarrow \gamma, Z^0 \quad \text{and} \quad W_\mu^1, W_\mu^2 \rightarrow W^+, W^-$$

Fermions acquire mass through Yukawa couplings to Higgs



The SM as an effective field theory

- Weinberg's "folk theorem":** If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties.
 (S. Weinberg, "Phenomenological Lagrangians," *Physica A*, 96, 327, 1979; "Effective Field Theory, Past and Future," [arXiv:0908.1964])
 For more information, see H.H. Zhang's lectures on EFT.

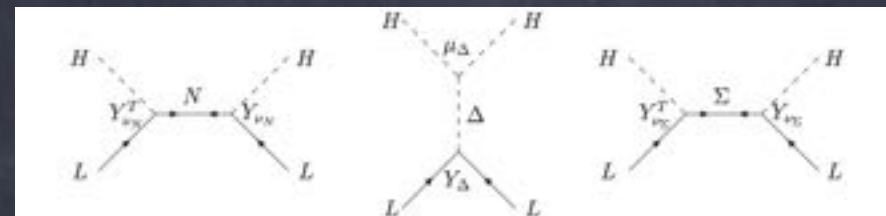
- SM effective field theory (SMEFT):**

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \dots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4$$

- d=5 Weinberg's operator:** After SSB, neutrinos obtain masses!

$$\mathcal{L}_{d=5} = \frac{c_{ij}}{\Lambda_L} E_L^{i\alpha T} C \epsilon_{\alpha\beta} H^\beta H^\lambda \epsilon_{\lambda\delta} E_L^{j\delta} + \text{h.c.} \Rightarrow m_{\nu ij} = -\frac{v^2}{\Lambda_L} c_{ij}.$$

Can be obtained from tree-level see-saw



For neutrino masses and mixings related topics, see 热衣木阿吉 & Z.H. Zhao's lectures

Successes of the SM

- All known “elementary” particles are involved in the SM;
- ONLY 18 parameters;
- ALMOST ALL known high energy phenomena (except neutrino oscillations) can be described qualitatively and quantitatively;
 - Classifications of hadrons: approximate flavor symmetry in QCD, chiral symmetry breaking, exotic hadrons (predicted by QCD but yet not fully understood) ...
 - Hard scattering processes such as DIS, Drell–Yan and jets: Factorization works well thanks to the asymptotic freedom of QCD ...
 - Charge current and neutral current processes;
 - P/CP violations;
 - Neutral K/B/D meson mixing;
 - Decays and CP asymmetries of K/B/D decays;
 - FCNC processes, lepton universalities

see lectures in this summer school for details

Experimental tests of the SM

- Most of tests from accelerator related experiments (collider physics)!
For collider physics, see L.Wu's lectures
 - The gauge sector: LEP, SLC, Tevatron
Gauge couplings, gauge structures, generations, ...
 - The flavor sector: B factories (CLEO/BaBar/Belle/LHCb/Belle-II)
Decays&CPV \Rightarrow Determination of CKM matrix-elements: size and phases
 - The EWSB sector (Higgs related): LHC/CEPC/ILC
Discovery of Higgs, gauge couplings, Yukawa couplings, self-couplings
 - The neutrino-mass sector: neutrino factories
Oscillations/CP violation \Rightarrow masses and mixings
- QCD plays a very essential role in most of above tests!
see C.F. Qiao's lectures

The most successful example of the SM

Anomalous magnetic moment of leptons

$$\vec{\mu}_\ell = g_\ell \left(\frac{q}{2m_\ell} \right) \vec{s} \quad \text{where} \quad g_\ell = 2(1 + a_\ell)$$

At order of α , $a_\ell = \frac{\alpha}{2\pi} \approx 0.0011614$

For electron:

$$a_e(\text{Exp}) = 1159652180.73(28) \times 10^{-12}$$

$$a_e(\text{SM} : \alpha(\text{Rb})) = 1159652182.037(720)(11)(12) \times 10^{-12},$$

$$a_e(\text{SM} : \alpha(\text{Cs})) = 1159652181.606(229)(11)(12) \times 10^{-12},$$

$$a_e(\text{Exp}) - a_e(\text{SM}) \approx 1 \times 10^{-12} \quad \text{!!!!!!!!!!!!!!}$$

For muon:

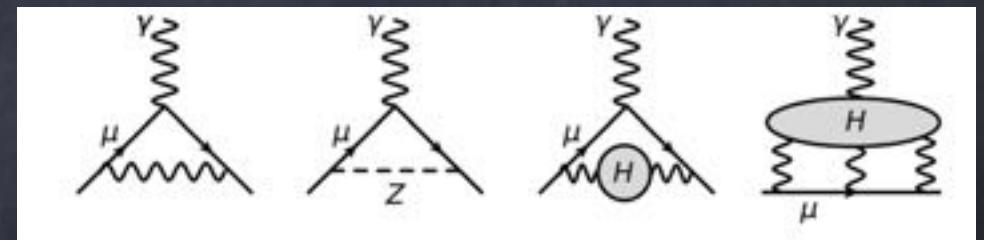
$$a_\mu(\text{Exp} : \text{BNL} + \text{FNAL}) = 116592061(41) \times 10^{-11} (0.35 \text{ppm})$$

$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11} (0.37 \text{ppm})$$

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

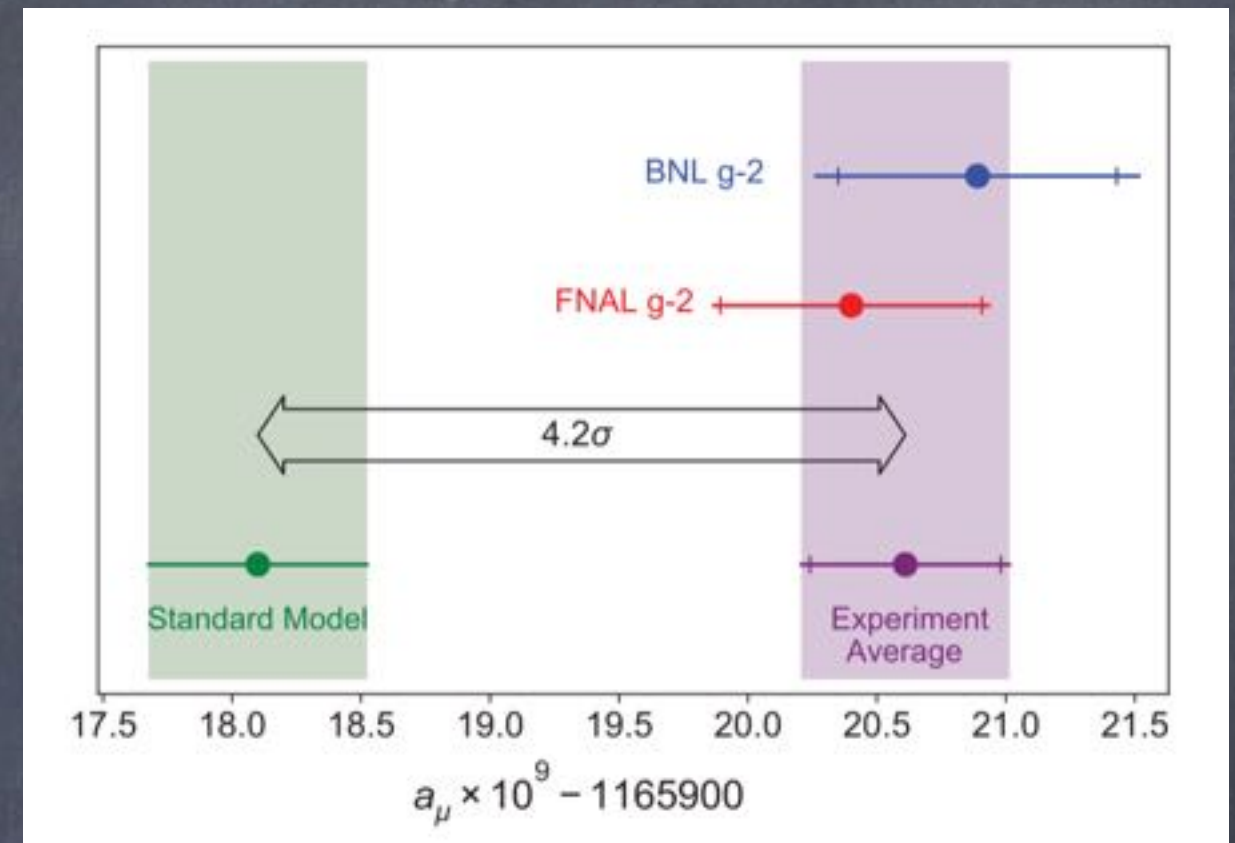
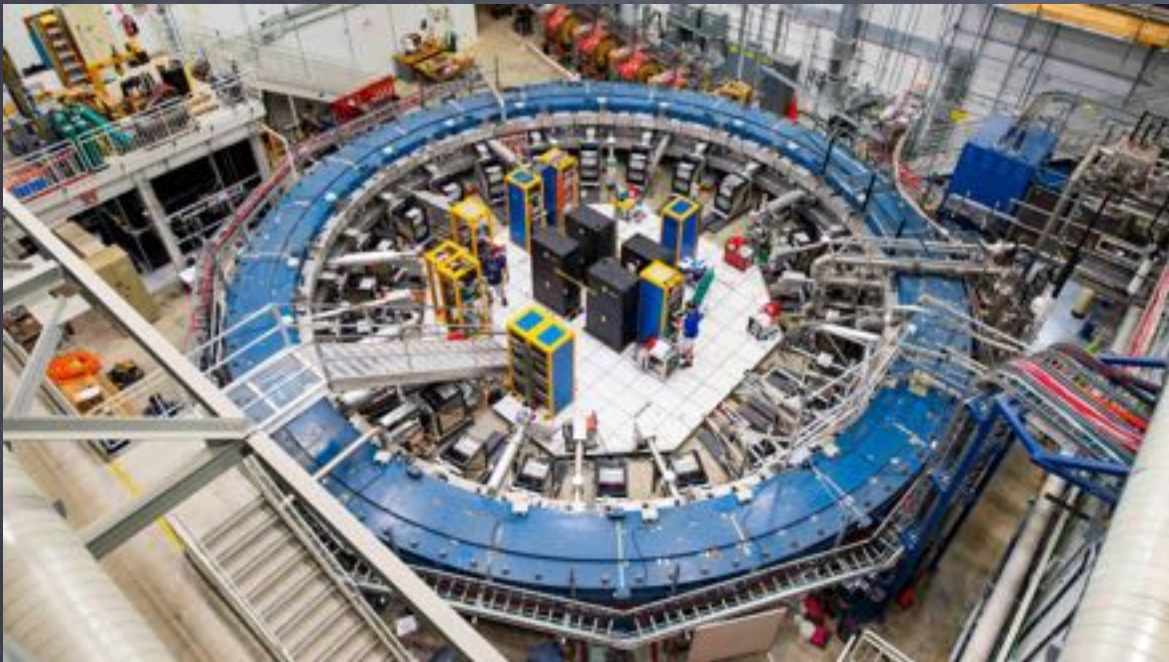


The headstone of Julian Schwinger at Mt. Auburn Cemetery in Cambridge, MA.

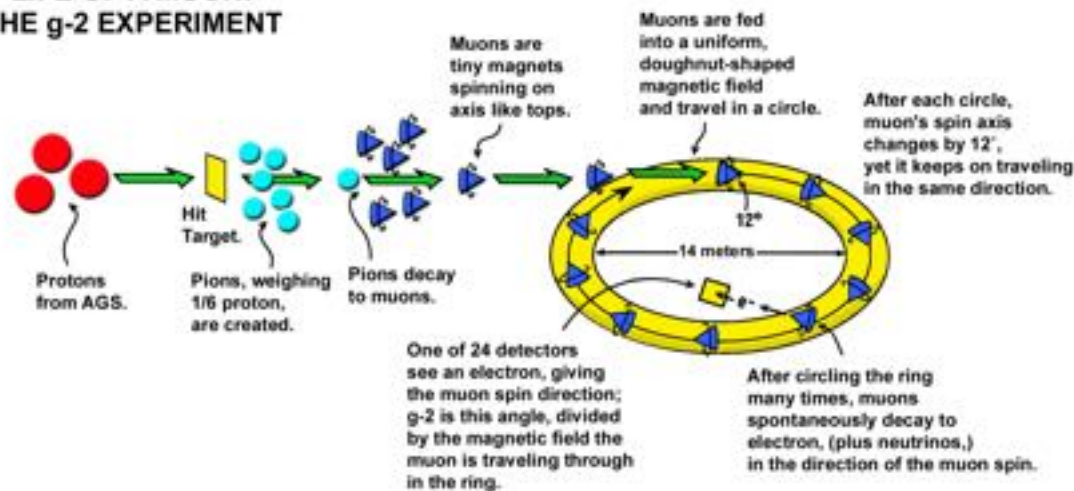


Efforts from experimentalists

BNL E821+FNAL muon g-2 experiments



LIFE OF A MUON: THE g-2 EXPERIMENT



$$\vec{\omega}_a = -\frac{Qe}{m} \left[a_\mu \vec{B} - \left(a_\mu - \left(\frac{mc}{p} \right)^2 \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] = -\frac{Qe}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]. \quad (3.10)$$

For the “magic” momentum $p_{\text{magic}} = m/\sqrt{a} \simeq 3.09 \text{ GeV}/c$ ($\gamma_{\text{magic}} = 29.3$), the second term vanishes, and the electric field does not contribute to the spin motion *relative* to the momentum.² If $g = 2$, then $a_\mu = 0$ and the spin would follow the momentum, turning at the cyclotron frequency.

$$a_\mu(\text{FNAL}) = 116592040(54) \times 10^{-11}$$

$$a_\mu(\text{BNL}) = 116592089(63) \times 10^{-11}$$

$$a_\mu(\text{Exp}) = 116592061(41) \times 10^{-11} (0.35 \text{ ppm})$$

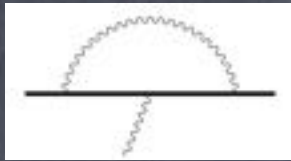
$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11} (0.37 \text{ ppm})$$

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

PRL 126, 141801 (2021)

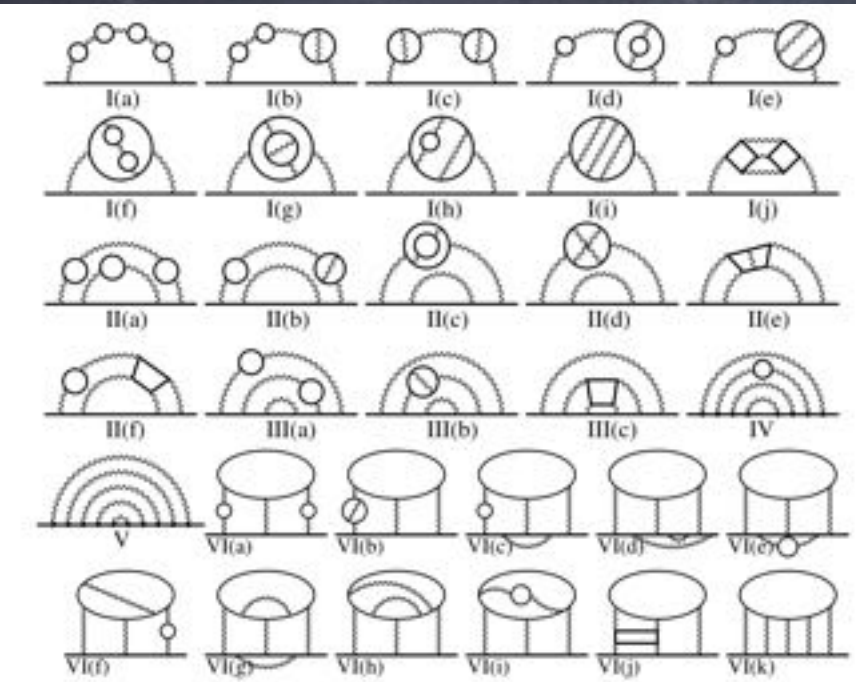
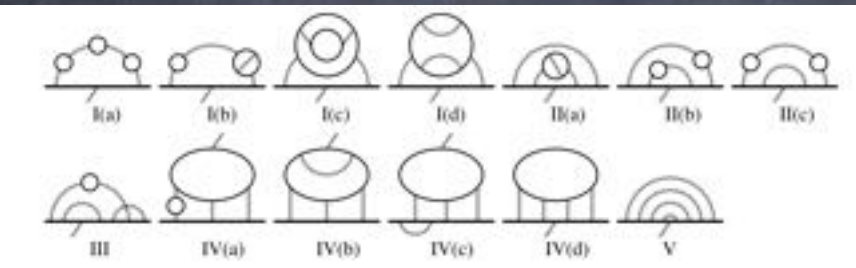
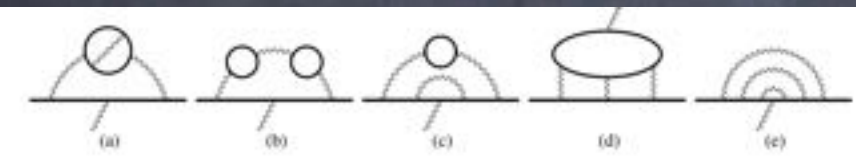
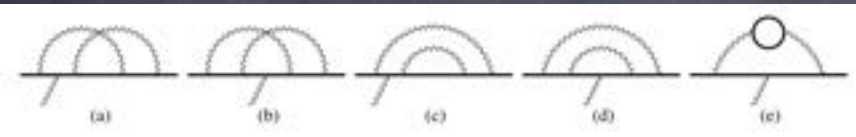
Efforts from theorists

QED:



EW:

$$a_{\mu}^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$$



$$a_{\mu}^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931(104) \times 10^{-11}.$$

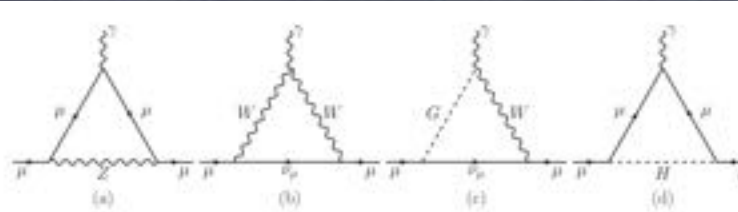


Fig. 100. One-loop Feynman diagrams contributing to a_{μ}^{EW} .

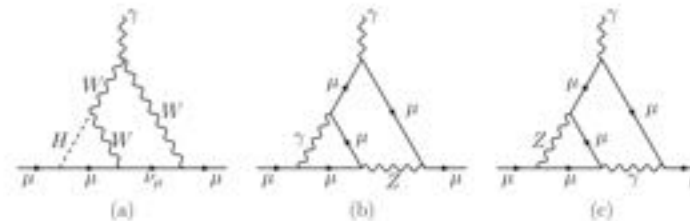


Fig. 101. Sample bosonic two-loop Feynman diagrams contributing to a_{μ}^{EW} .

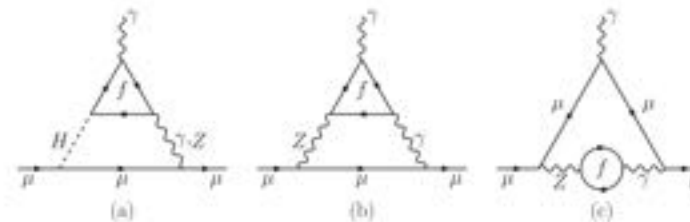
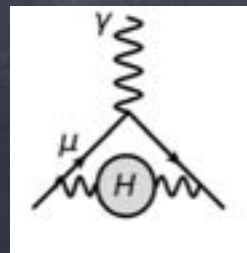
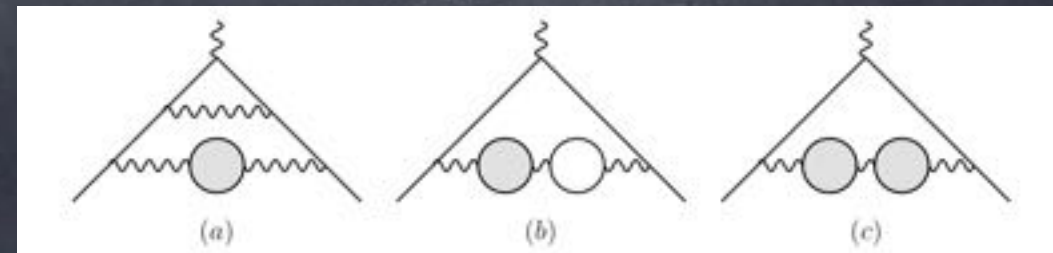


Fig. 102. Sample fermionic two-loop Feynman diagrams contributing to a_{μ}^{EW} .

HVP:



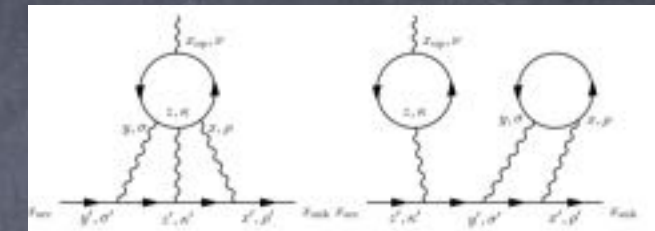
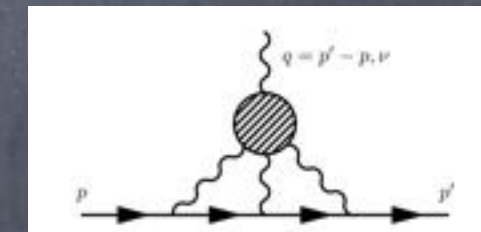
$$a_{\mu}^{\text{HVP, LO}} + a_{\mu}^{\text{HVP, NLO}} + a_{\mu}^{\text{HVP, NNLO}} = 6845(40) \times 10^{-11}$$



$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{HVP, LO}} + a_{\mu}^{\text{HVP, NLO}} + a_{\mu}^{\text{HVP, NNLO}} + a_{\mu}^{\text{HLbL}} + a_{\mu}^{\text{HLbL, NLO}} = 116\,591\,810(43) \times 10^{-11}.$$

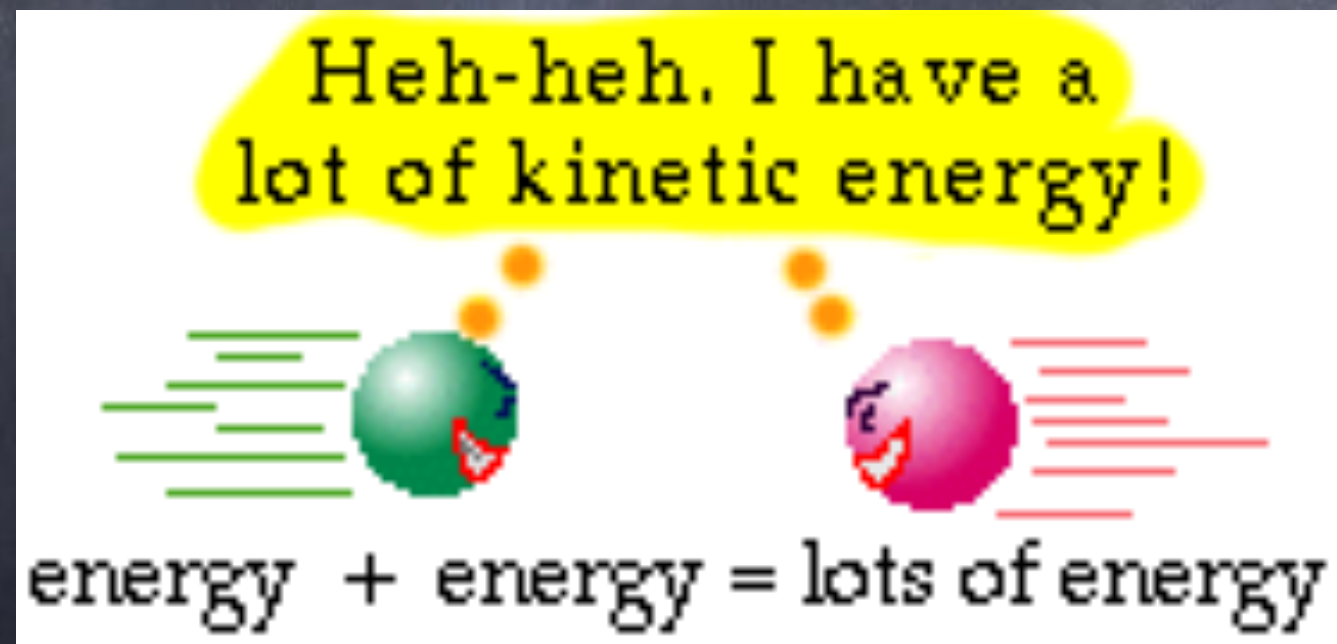
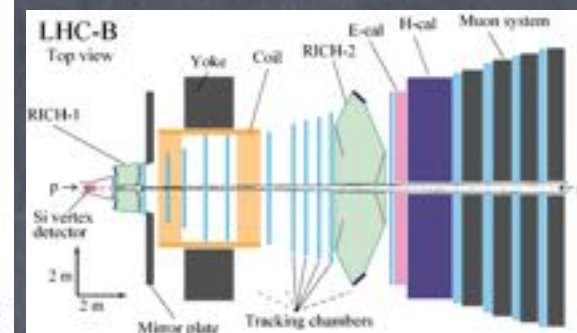
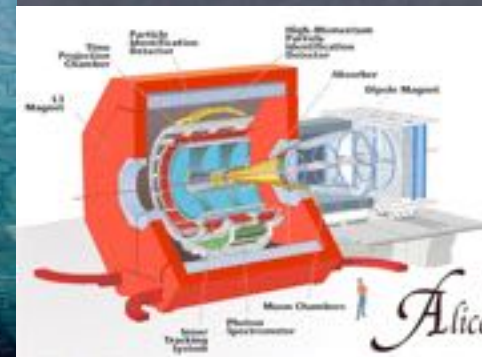
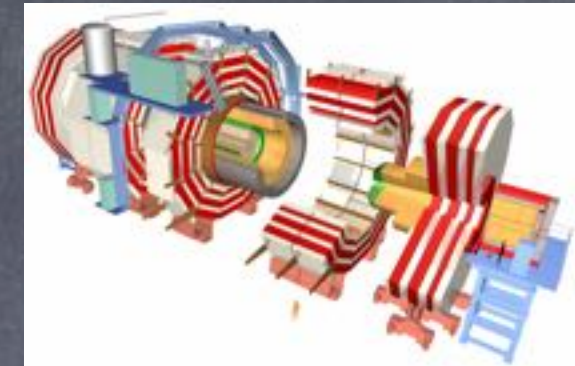
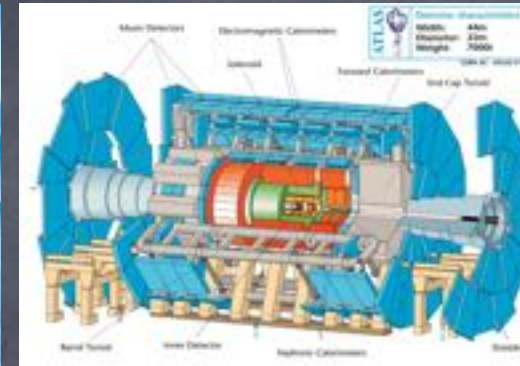
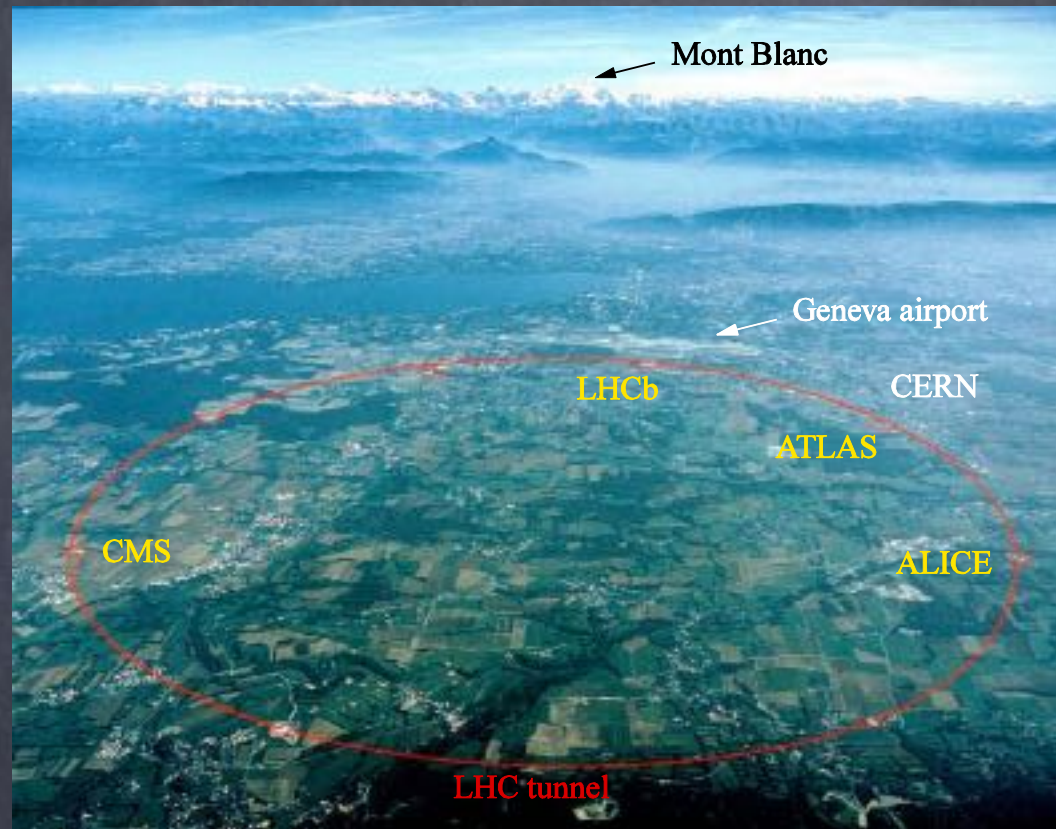


HLbL:



$$a_{\mu}^{\text{HLbL}}(\text{phenomenology} + \text{lattice QCD}) = 90(17) \times 10^{-11}$$

Large Hadron Collider (LHC)



Main channels to produce Higgs@LHC

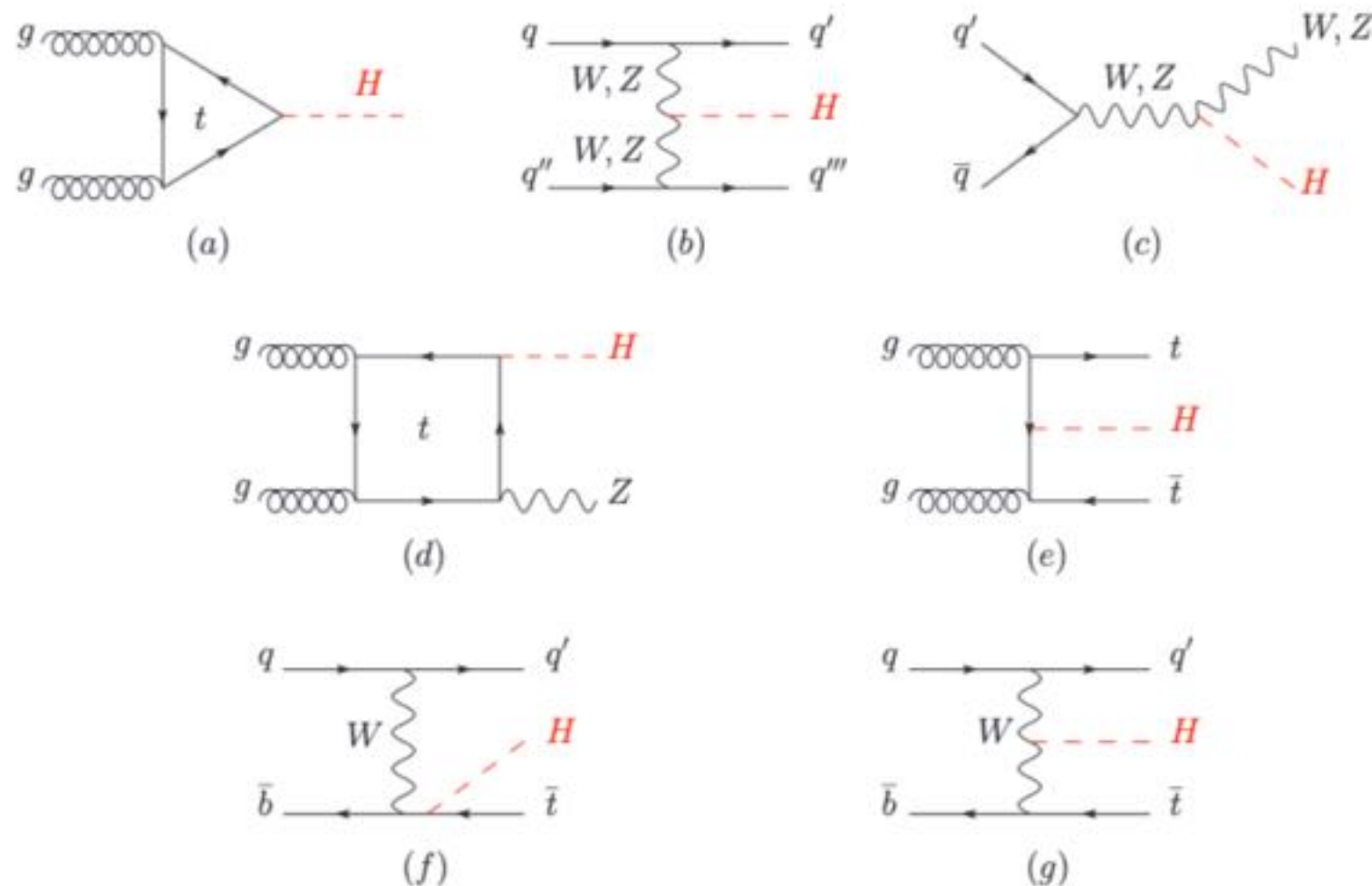
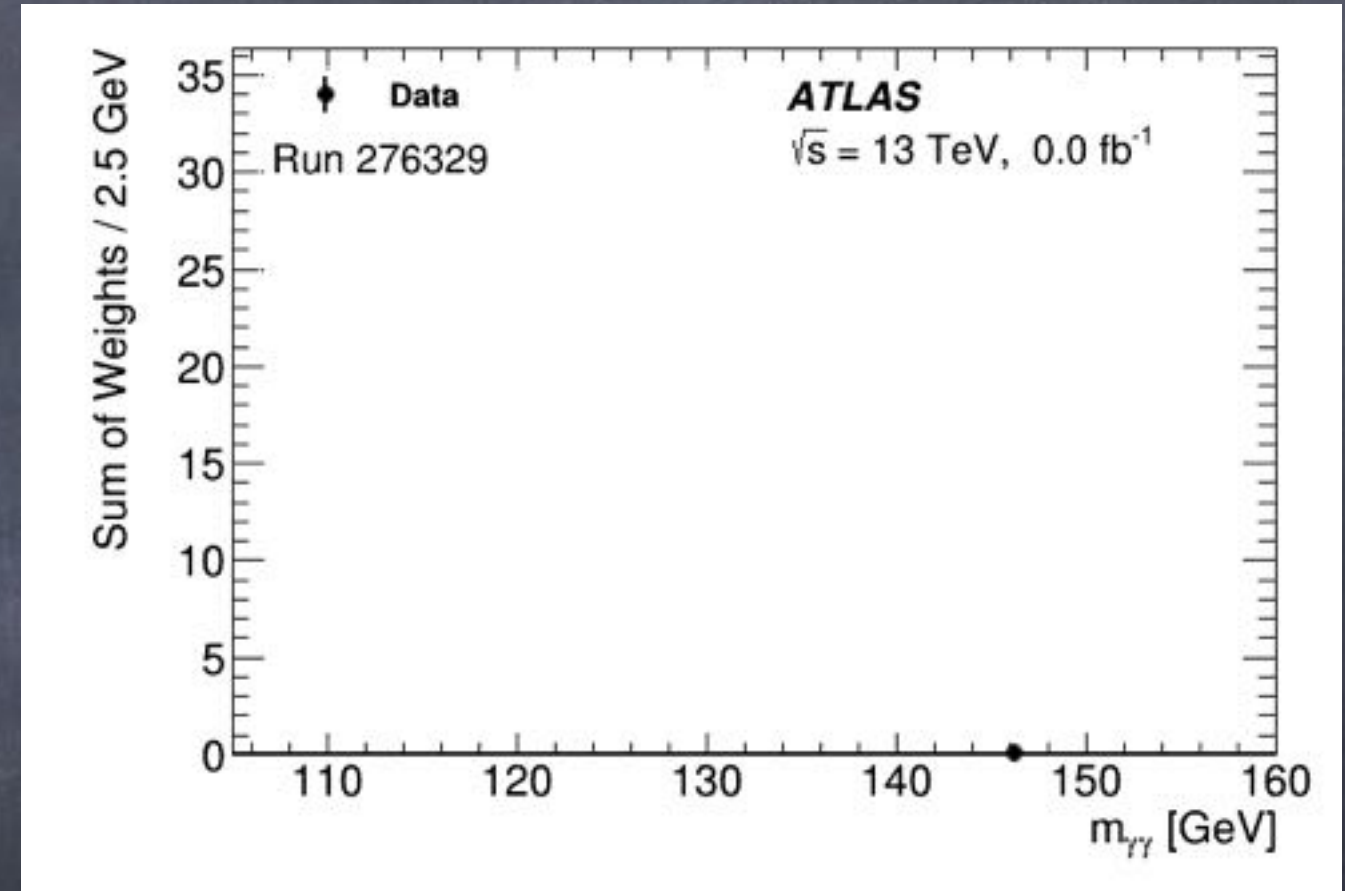
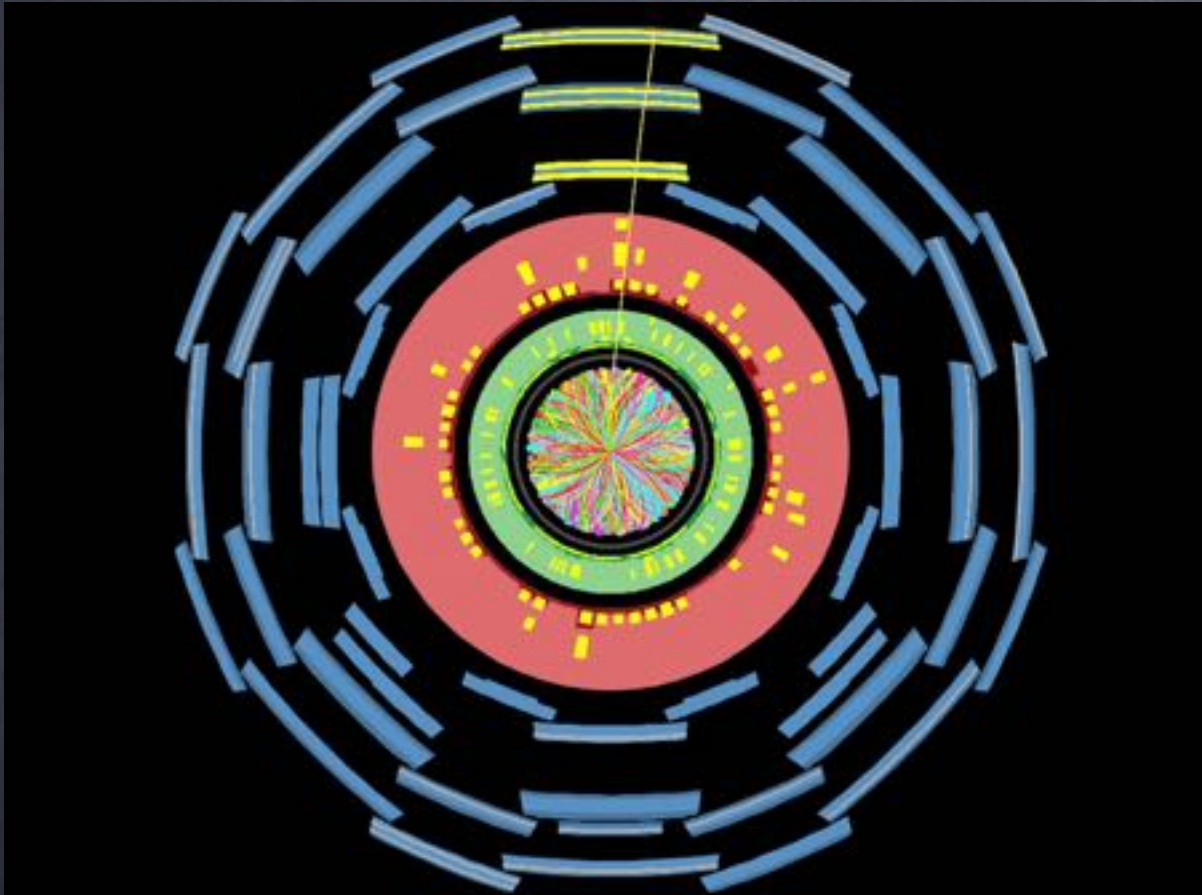


Figure 11.1: Main leading order Feynman diagrams contributing to the Higgs boson production in (a) gluon fusion, (b) Vector-boson fusion, (c) Higgs-strahlung (or associated production with a gauge boson at tree level from a quark-quark interaction), (d) associated production with a gauge boson (at loop level from a gluon-gluon interaction), (e) associated production with a pair of top quarks (there is a similar diagram for the associated production with a pair of bottom quarks), (f-g) production in association with a single top quark

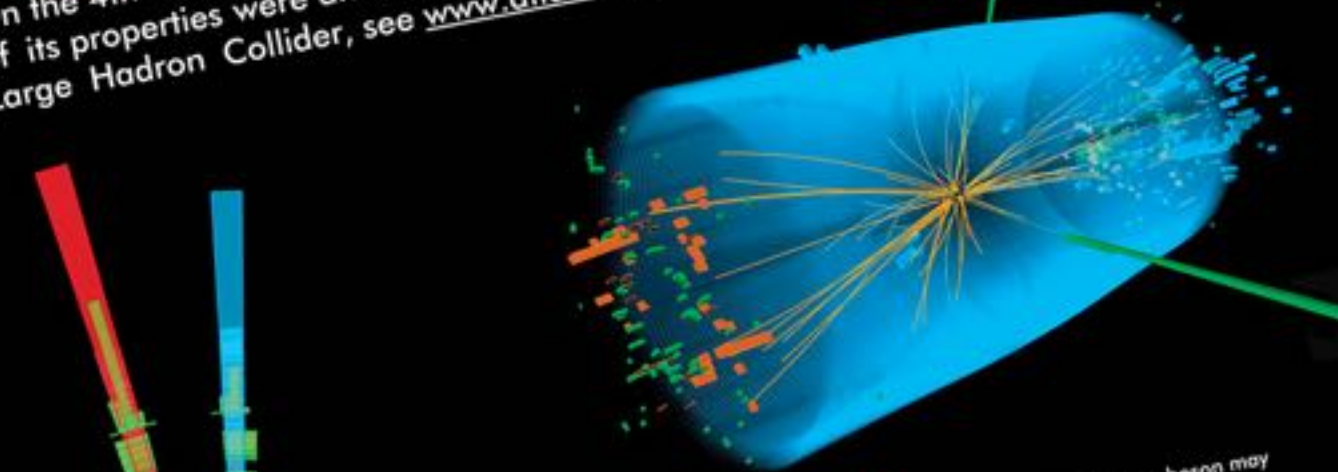
Discover Higgs@LHC



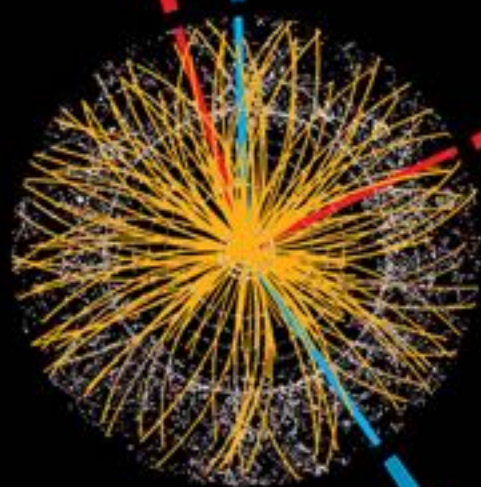
The Higgs Boson Born on the 4th of July

What gives masses to fundamental particles such as quarks and electrons?

On the 4th of July 2012, new results in the search for the Higgs boson and studies of its properties were announced by the ATLAS and CMS Experiments at the Large Hadron Collider, see www.atlas.ch and cms.cern.ch.



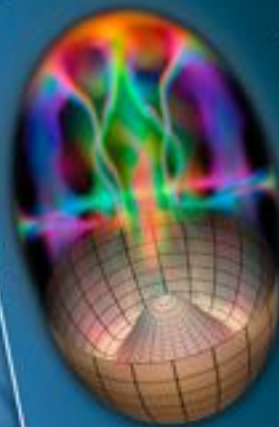
In this collision in the CMS detector, a Higgs boson may have been produced along with other particles. This possible Higgs boson was identified from the decay products of two photons, shown by the green bars.



In this collision in the ATLAS detector, a Higgs boson may have been produced along with other particles. This possible Higgs boson was identified from the decay products of two electrons and two anti-electrons (positrons), shown by the red and blue bars.

The Higgs Field and the Boson

All of space is permeated by a field, the Higgs field.



The Higgs field, responsible for the mass of fundamental particles, fills the universe.

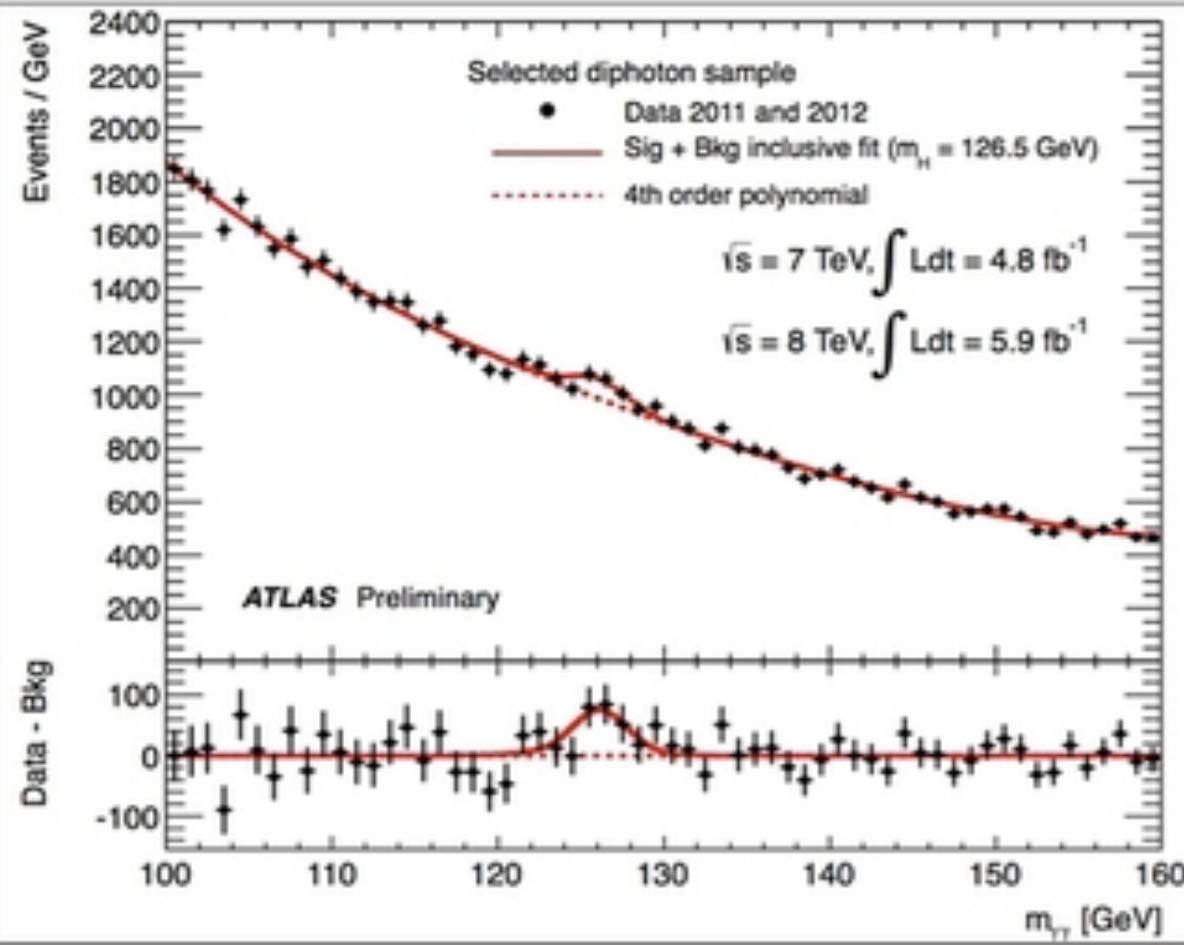
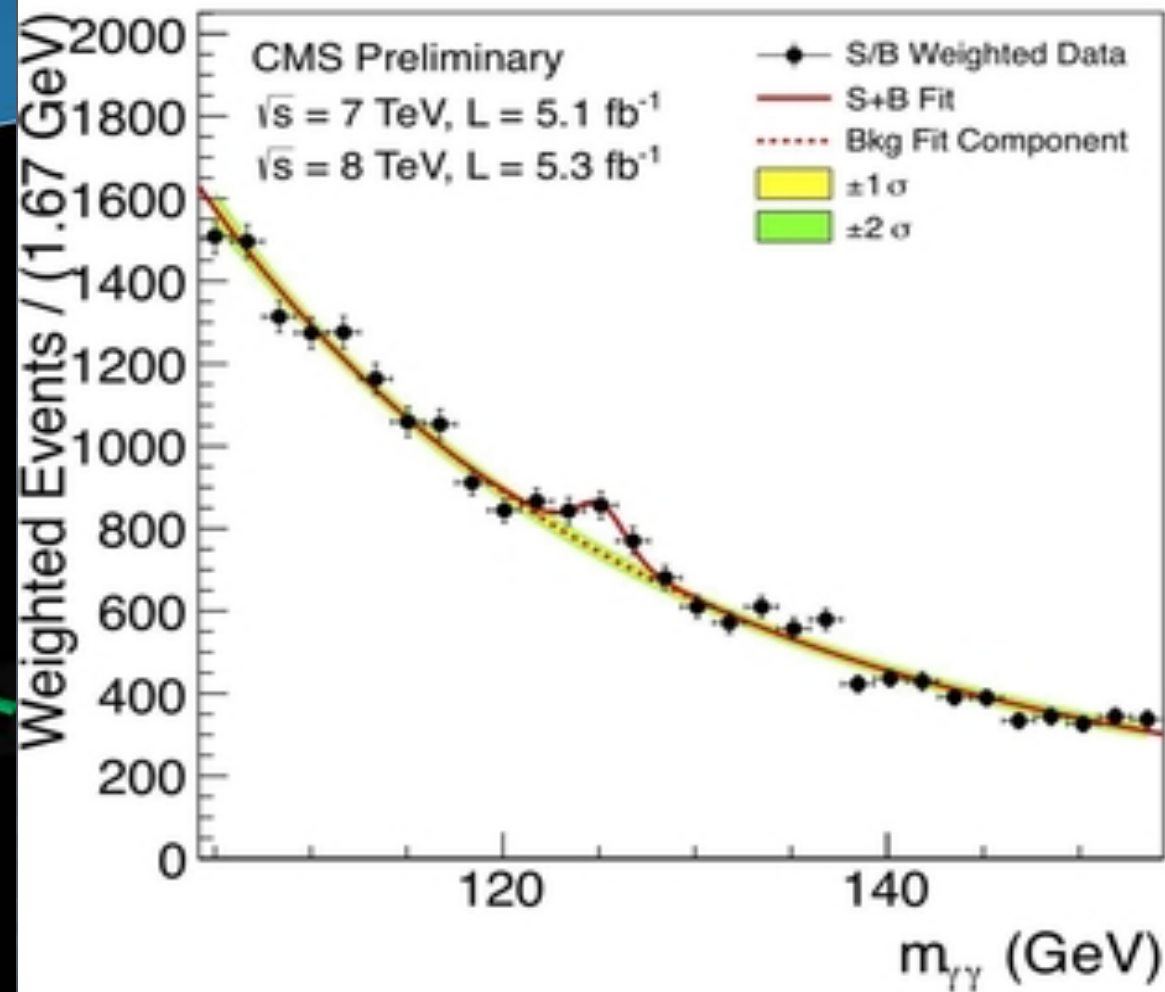
This image shows a representation of the field.

Quantum theory says that all fields have particles associated with them, so... in this case... a Higgs boson.

Learn more at
ParticleAdventure.org
and at CPEPphysics.org



This chart has been made possible by the generous support of:
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Lawrence Berkeley National Laboratory
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Main production/decay channels of Higgs

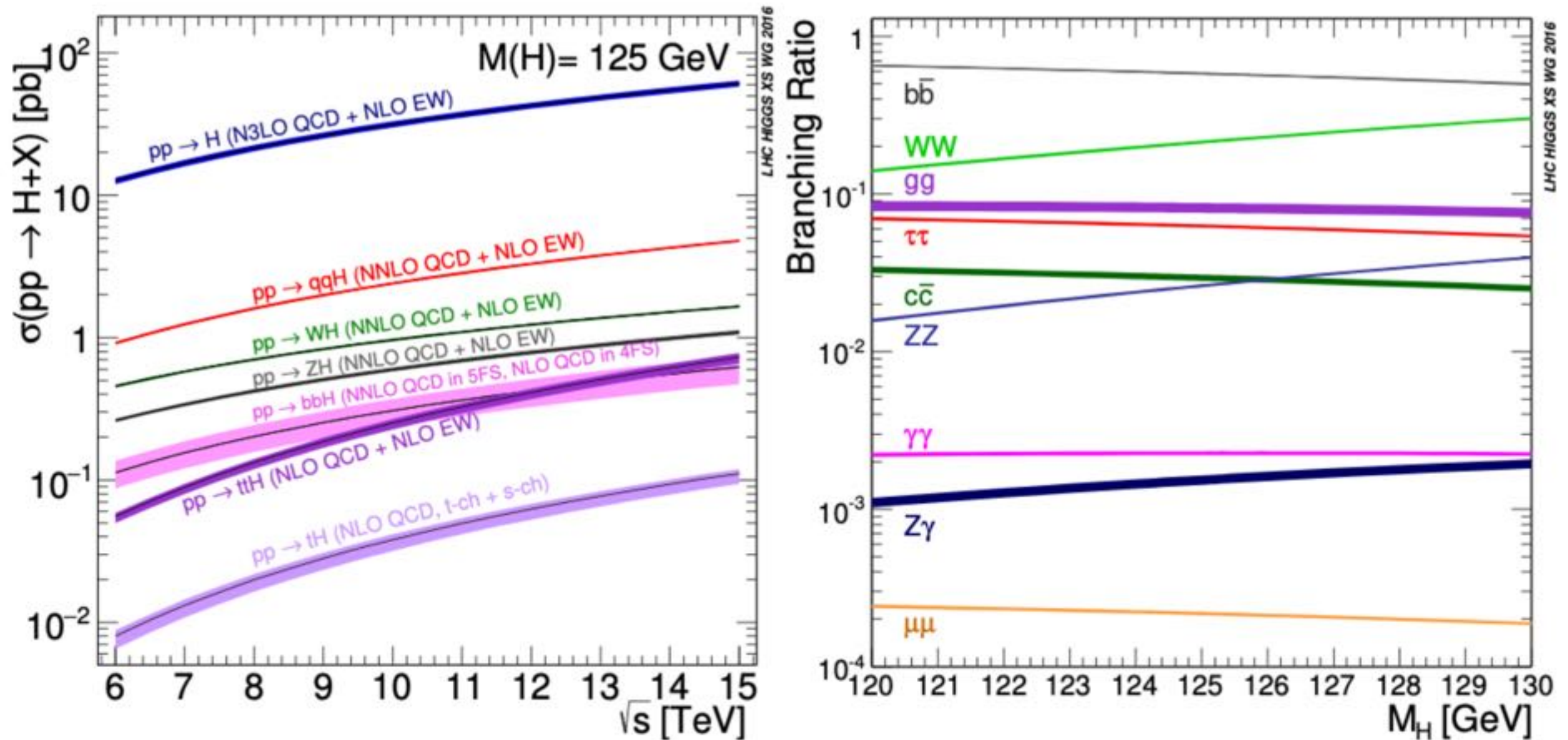


Figure 11.2: (Left) The SM Higgs boson production cross sections as a function of the center of mass energy, \sqrt{s} , for pp collisions [45]. The VBF process is indicated here as qqH . The theoretical uncertainties are indicated as bands. (Right) The branching ratios for the main decays of the SM Higgs boson near $m_H = 125$ GeV [43, 44]. The theoretical uncertainties are indicated as bands.

Main decay channels of Higgs

Table 11.3: The branching ratios and the relative uncertainty [43,44] for a SM Higgs boson with $m_H = 125$ GeV.

Decay channel	Branching ratio	Rel. uncertainty
$H \rightarrow \gamma\gamma$	2.27×10^{-3}	2.1%
$H \rightarrow ZZ$	2.62×10^{-2}	$\pm 1.5\%$
$H \rightarrow W^+W^-$	2.14×10^{-1}	$\pm 1.5\%$
$H \rightarrow \tau^+\tau^-$	6.27×10^{-2}	$\pm 1.6\%$
$H \rightarrow b\bar{b}$	5.82×10^{-1}	+1.2% -1.3%
$H \rightarrow c\bar{c}$	2.89×10^{-2}	+5.5% -2.0%
$H \rightarrow Z\gamma$	1.53×10^{-3}	$\pm 5.8\%$
$H \rightarrow \mu^+\mu^-$	2.18×10^{-4}	$\pm 1.7\%$

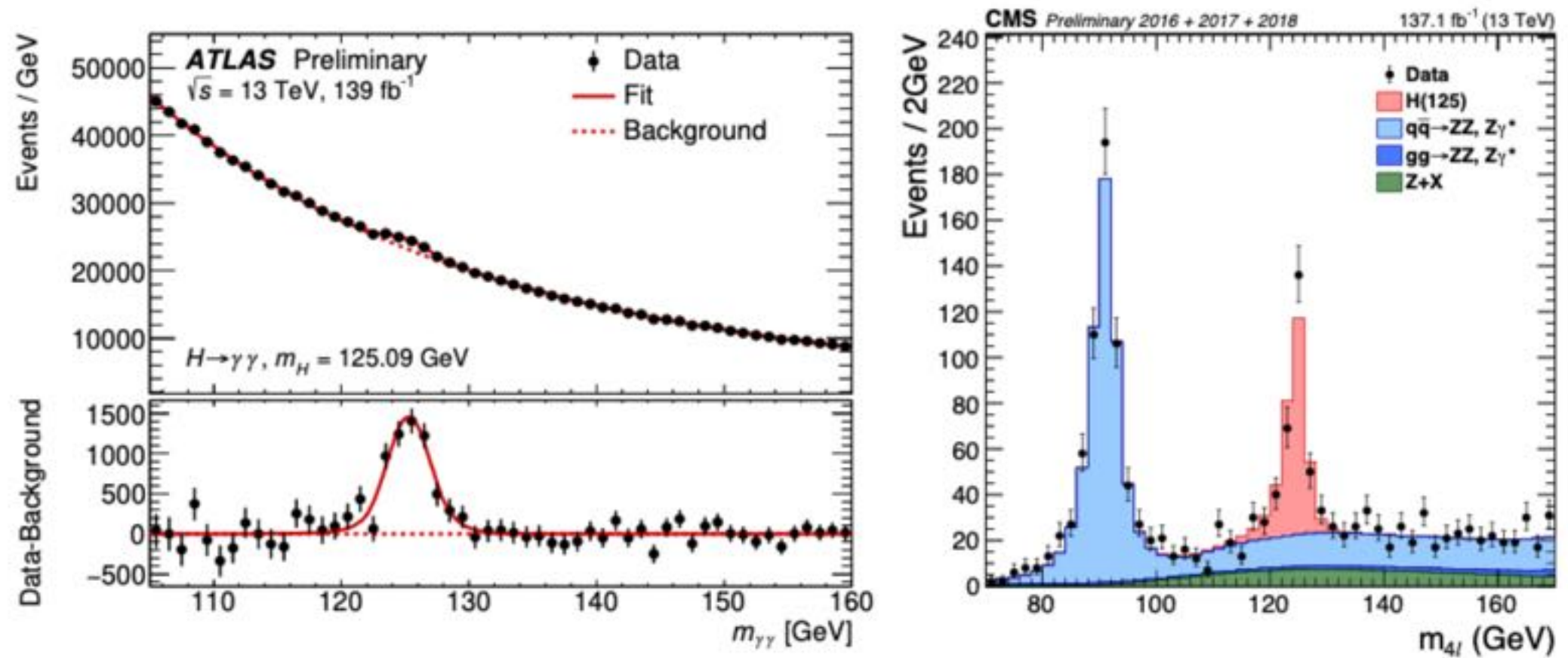


Figure 11.3: (Left) The invariant mass distribution of diphoton candidates, with each event weighted by the ratio of signal-to-background in each event category, observed by ATLAS [125] at Run 2. The residuals of the data with respect to the fitted background are displayed in the lower panel. (Right) The $m_{4\ell}$ distribution from CMS [126] Run 2 data.

Status of Higgs mass measurements

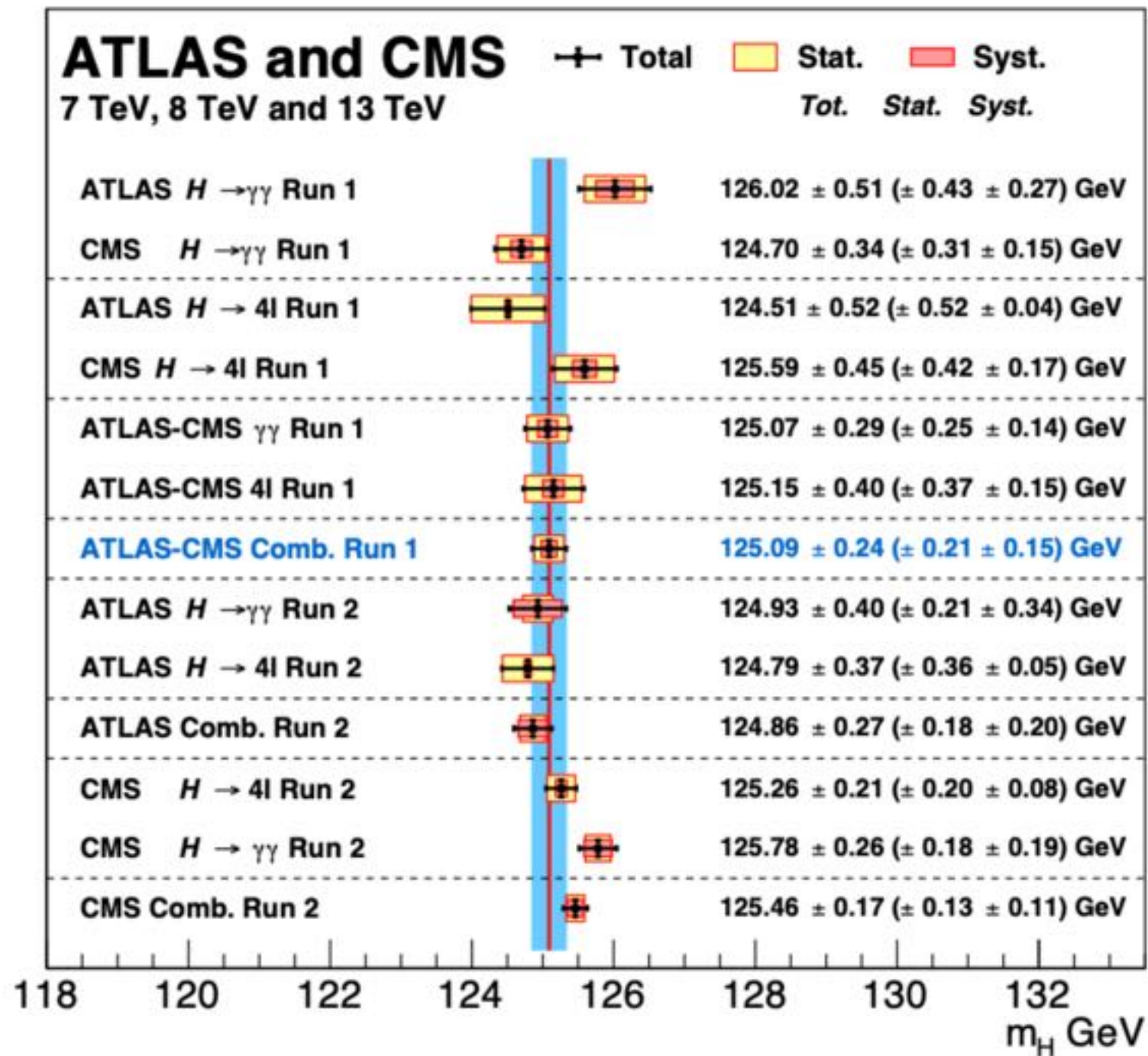


Figure 11.4: Summary of the CMS and ATLAS mass measurements in the $\gamma\gamma$ and ZZ channels in Run 1 and Run 2.

Status of experimental measurements normalized to the SM predictions

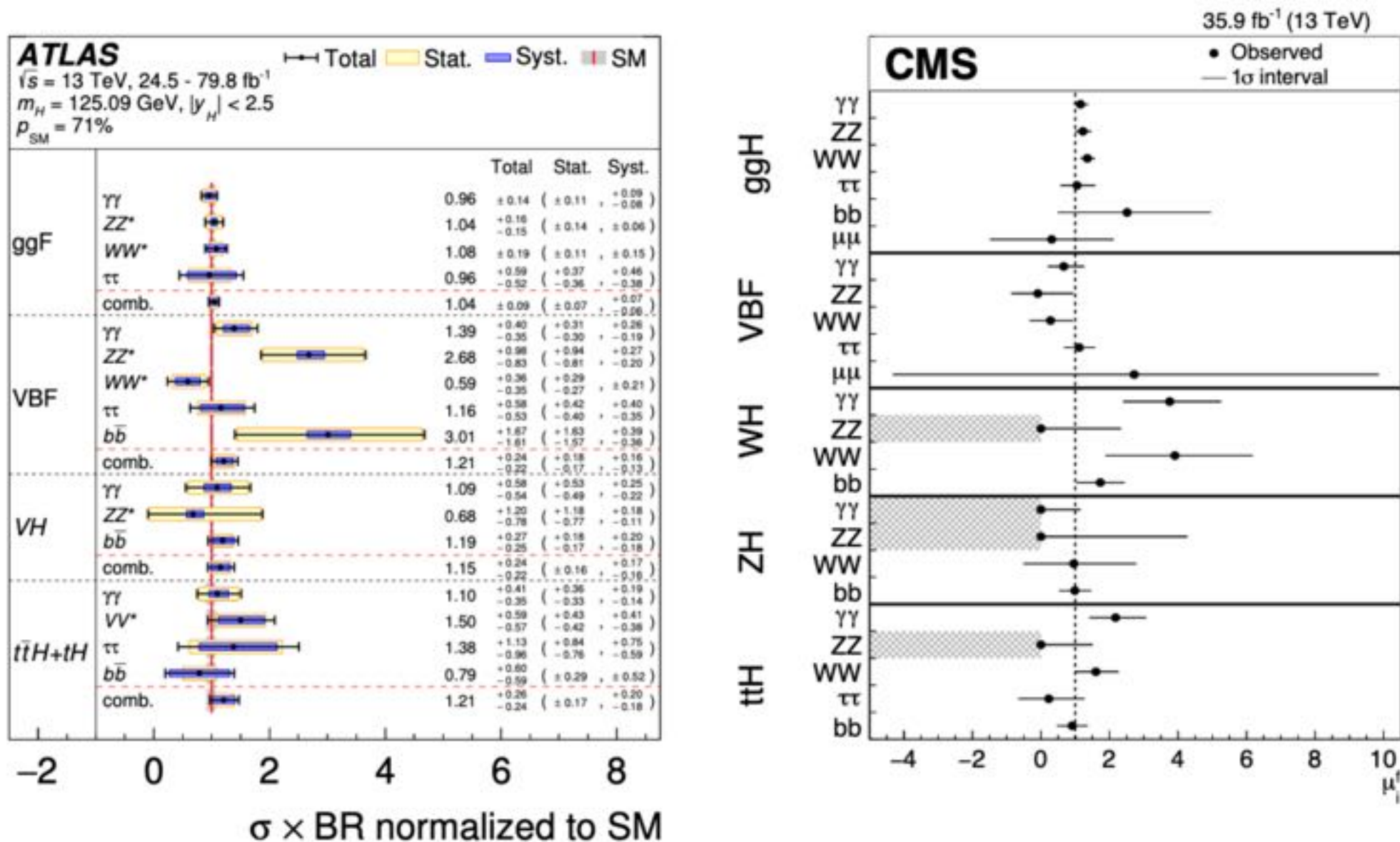


Figure 11.8: Combined measurements of the products $\sigma \cdot \text{BR}$, normalised to the SM predictions, for the five main production and five main decay modes. The hatched combinations require more data for a meaningful confidence interval to be provided.

Various constraints on the unitary triangle

- Neutral K/B meson mixings and B meson decays
 - B factories, LHCb experiments
- More information from lectures by Prof. C.D. Lü in this school

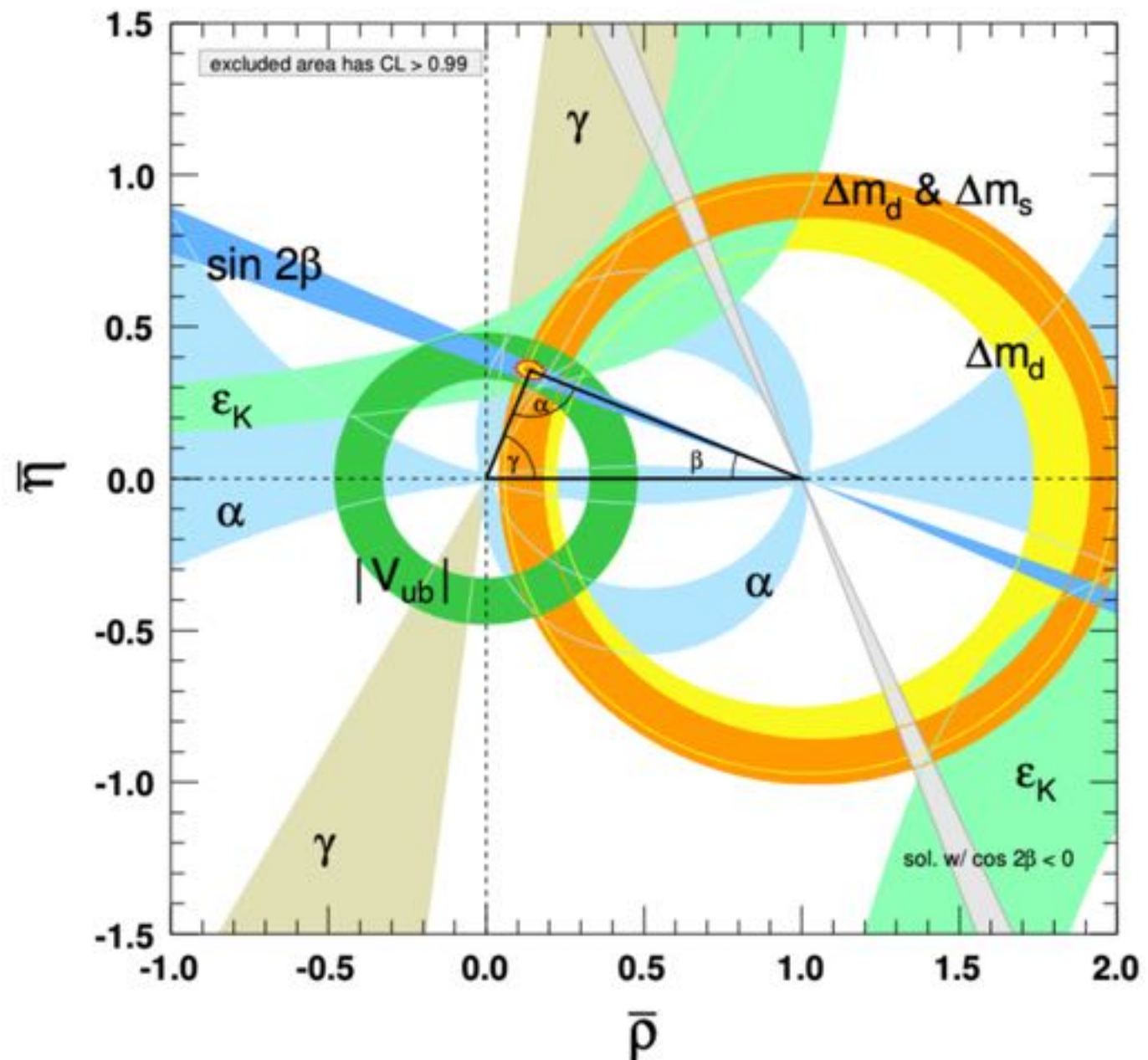


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 99% CL.

Unsatisfying issues of the SM

• Theoretical side

- Why $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$?

Gauge couplings unification? Grand unification theory?

- Why three generations? What's the origin of the patterns of quark/lepton masses and mixings? What's the property/origin of the neutrino masses?

Flavor symmetries/See-saw/String theory?

- Is the Higgs boson discovered at LHC the ONLY guy to be responsible for the electro-weak symmetry breaking (EWSB)?

Two Higgs doublets model/SUSY/Composite Higgs/Extra Dimension/Little Higgs/Technicolor/...

- Hierarchy problem/Fine tuning problem/Naturalness

- Strong CP problem ...

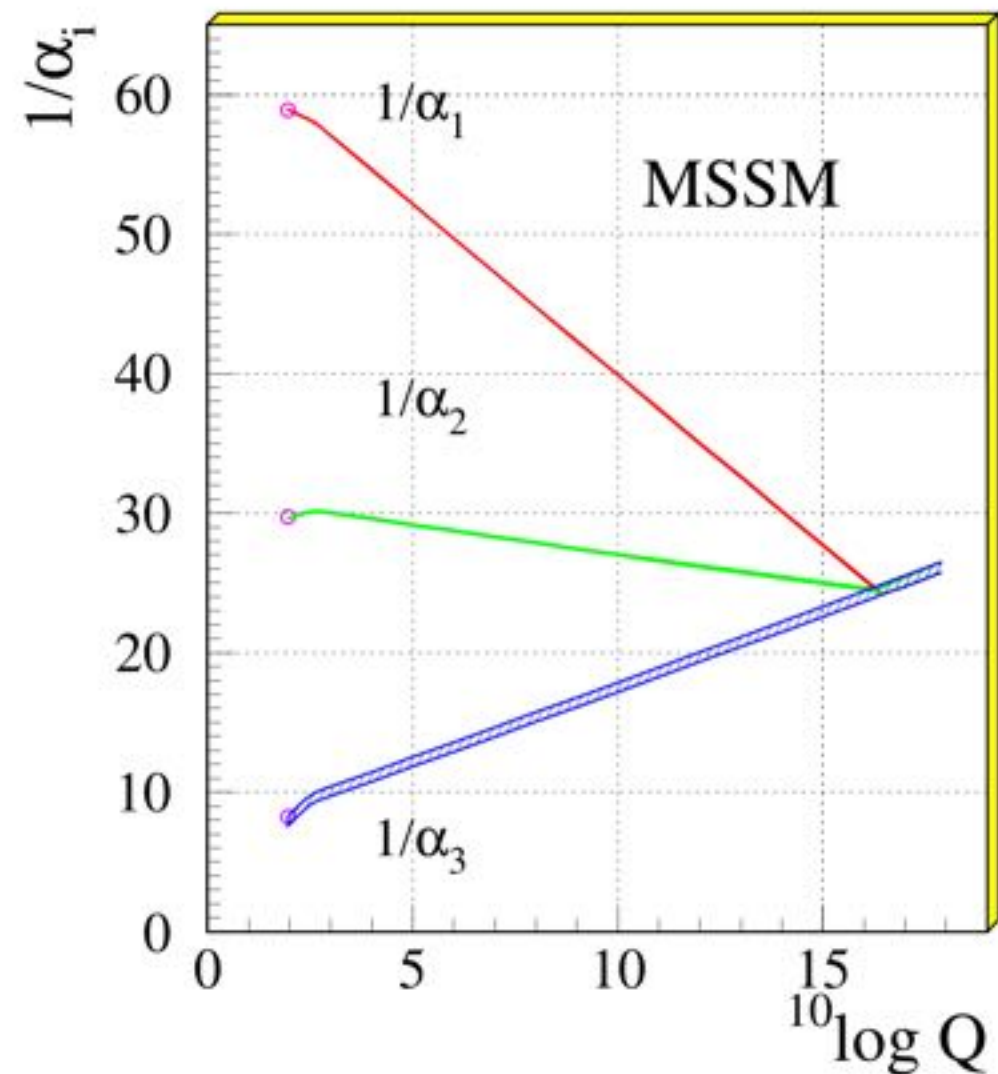
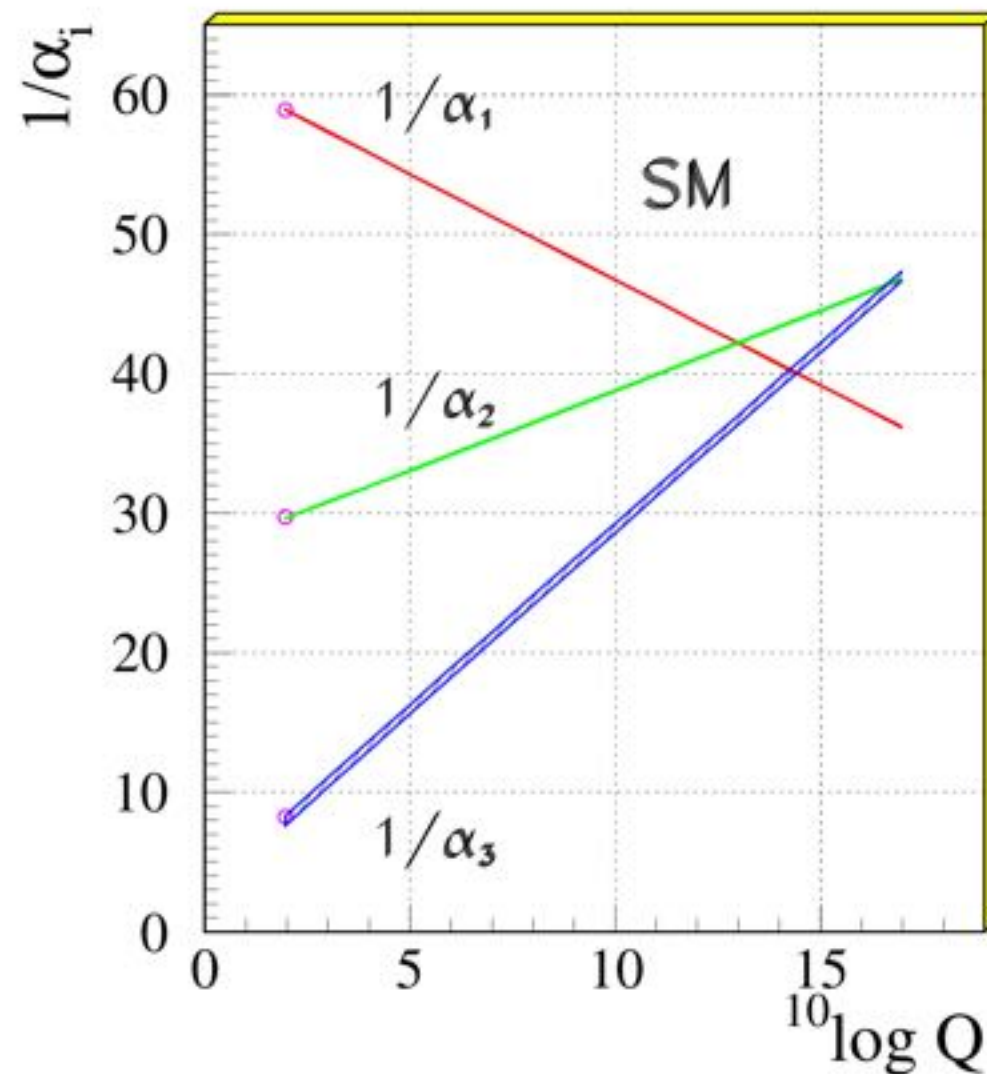
• Experimental/observational data (most from astronomy/cosmology)

Dark matter/dark energy/inflation/matter-anti-matter asymmetry

See lectures given by Q.Wang, Y.G.Gong, Z.Z.Xianyu, B.Q.Ma, B.Zhu, J.J.Cao, Z.H. Zhao etc in this school

Gauge couplings running in the SM/MSSM

Unification of the Coupling Constants in the SM and the minimal MSSM



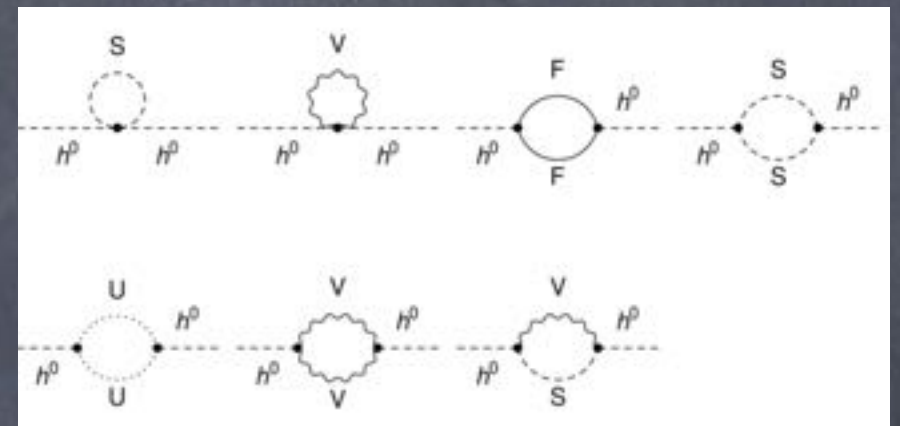
Within the SM, no gauge coupling unification! SUSY can help!
see X.G.Wu's lectures for RG running

Hierarchy problem

- Radiative quadratic divergence in Higgs self-energy

$$M_H^2 = (M_H^0)^2 + \delta M_H^2, \quad \delta M_H^2 \propto \int \frac{d^4 k}{k^2} \sim \Lambda_{UV}^2$$

$$\delta M_H^2 = \frac{3\Lambda^2}{8\pi^2 v^2} [M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2]$$



- If $\Lambda_{UV} \sim \Lambda_{\text{Planck}} \sim 10^{19} \text{ GeV}$, $\delta M_H^2/M_H^2 \sim 10^{34}$;
 \Rightarrow Hierarchy problem/fine tuning problem/naturalness problem
- Many solutions: see next slide
- SUSY can help! The signs of the quadratic divergence from bosons and fermions are opposite! The super symmetry (SUSY) is a symmetry between bosons and fermions! Bosons $\xleftrightarrow{\text{SUSY}}$ Fermions
 see Z.Sun's lectures for more

Solutions to HP



1. Supersymmetry
2. Global symmetry
3. Discrete symmetry
4. Modular invariance
5. RS/Technicolor
6. LED/ 10^{32} xSM
7. LST/Clockwork
8. Classicalization
9. Disorder
10. Anthopics
11. Relaxation
12. NNaturalness
13. Crunching away
14. Conformal symmetry
15. Asymptotic fragility
16. Agravity
17. Lee-Wick Theory
18. Weak gravity conjecture
19. Non-commutative QFT
20. Weak scale from CC
21. AdS magic
22. Self-organized criticality
23. ...

With apologies for the many omissions...

Summary: SM is good, NP is better?

- The Standard Model is successful!
- We are not satisfied!
- Enjoy the school!

Thank you!

LHC Luminosity Profile

Michel Della Negra

$$L = 2 \times 10^{33} \quad L = 10^{34}$$

$$\text{SLHC: } L = 10^{35} \text{ (cm}^{-2}\text{s}^{-1}\text{)}$$

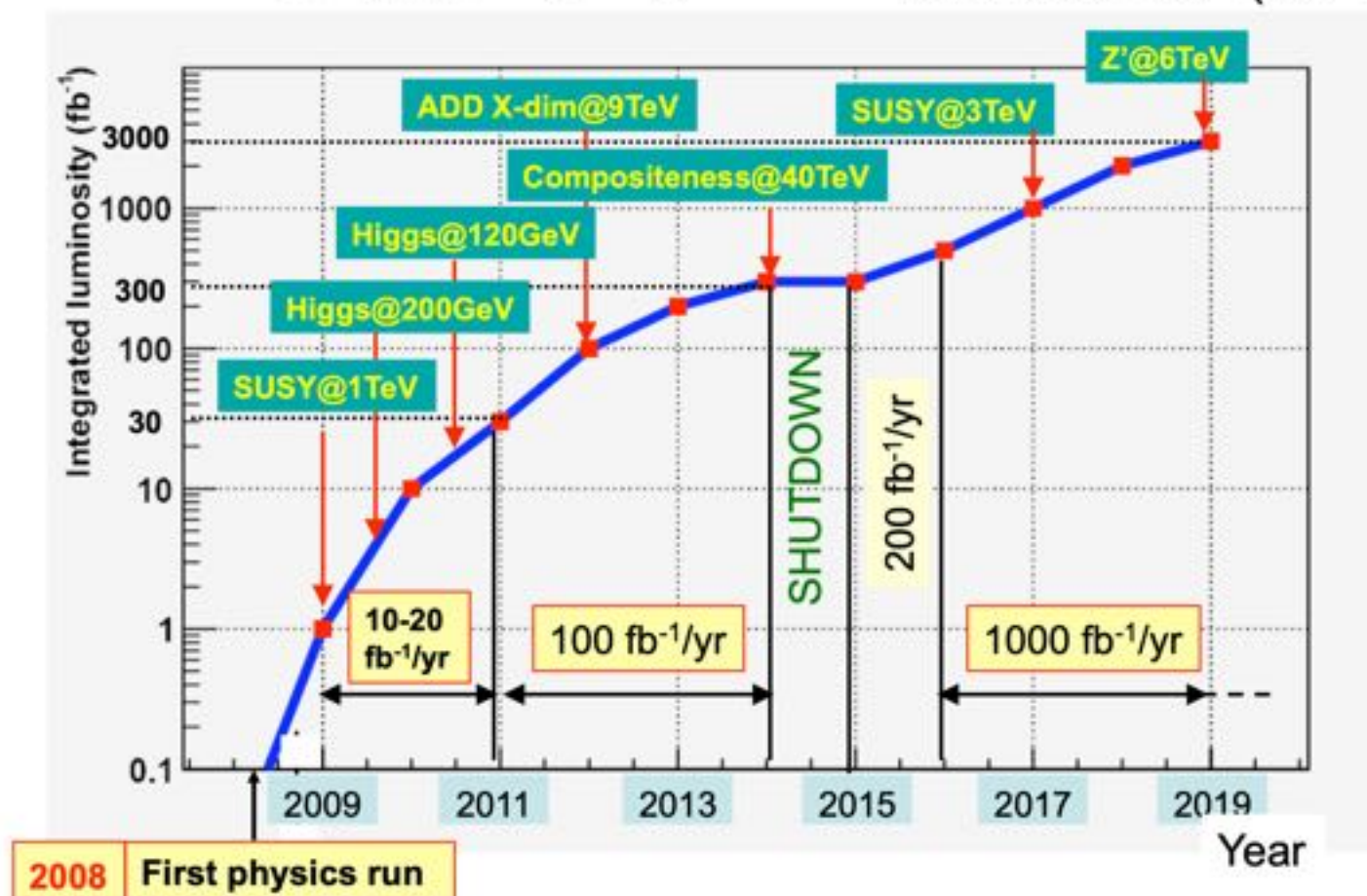


Table 11.4: The LHC *pp* collision centre-of-mass energies and delivered data samples.

Year	\sqrt{s} (TeV)	$\int L \cdot dt$ (fb ⁻¹)	Period
2010	7	0.04	Run 1
2011	7	6.1	Run 1
2012	8	23.3	Run 1
2015	13	4.2	Run 2
2016	13	40.8	Run 2
2017	13	50.2	Run 2
2018	13	60.6	Run 2

理想很魔
幻，现实很
艰难！

大家将上下而求索
路漫漫其修远兮